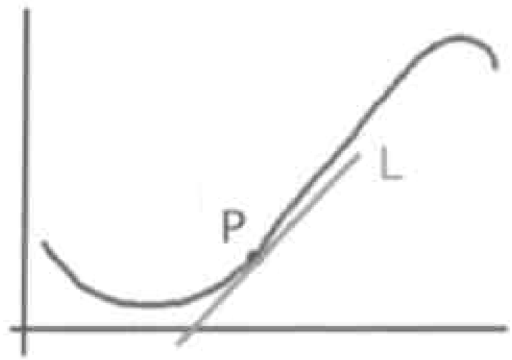
The Tangent and Velocity Problems

Section 2.1

Tangent Lines

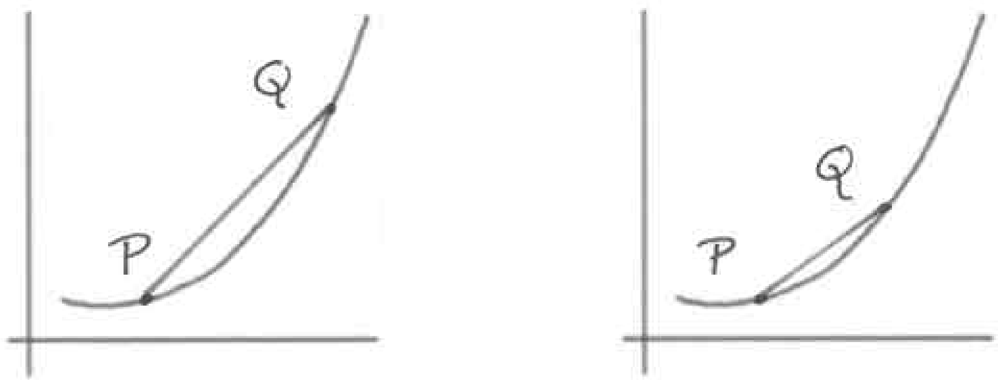
What do we mean by the tangent line to a curve at a point P?



- L should pass through P.
- As you walk along the curve, the direction of L should match the direction you face as you pass through P.

Slope of the Tangent Line

A secant PQ is a line segment joining P to a nearby point Q on the curve.



As Q -> P the slope m_{PQ} approaches the slope m of the tangent line.

$$m = \lim_{Q \to P} m_{PQ}$$

Example

The point $P = (2, 5)$ lies on $y = x^2 + 1$.

(a) Find the slope of the secant PQ for x-values approaching 2.
(b) Then predict the slope of the tangent line at P.
(c) Write an equation for the tangent line at P.

(a)

x	m_{PQ}	x	m_{PQ}
1.9	3.9000	2.1	4.1000
1.99	3.9900	2.01	4.0100
1.999	3.9990	2.001	4.0010
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
2^-	4	2^+	4

(b) From (a), we predict $m = 4$

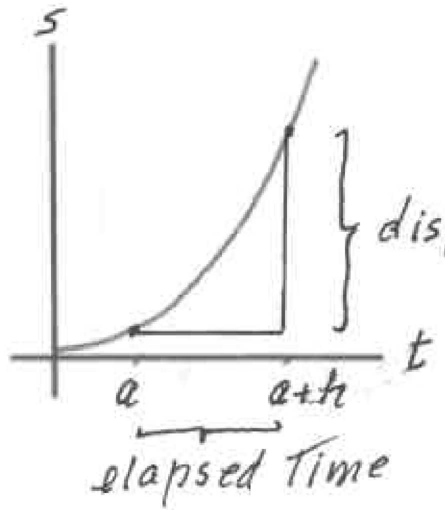
(c) Use point-slope form for tangent line: $\dfrac{y-5}{x-2} = 4$

Simplify: $y = 4x - 3$

Instantaneous Velocity

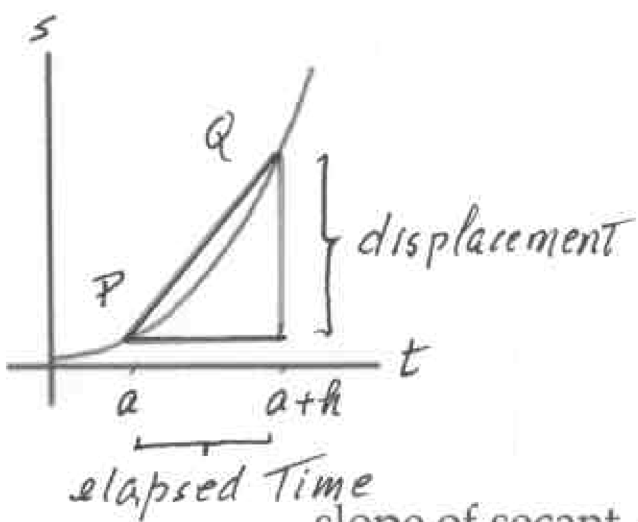
If s(t) is the position at time t of an object moving in a straight line, how can we define the (instantaneous) velocity at time a?

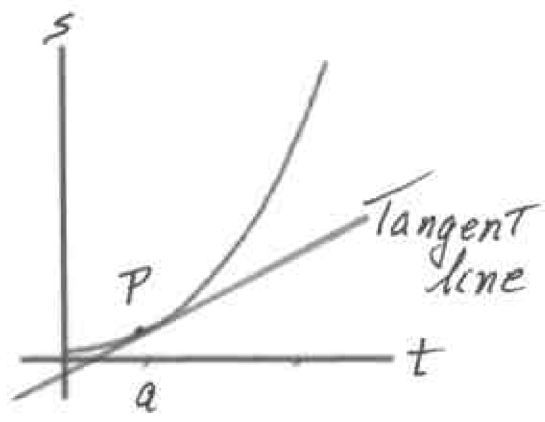
we know: average velocity = displacement / elapsed time



$$\begin{matrix}\text{instantaneous velocity}\\ \text{at time } a\end{matrix} = \lim_{h\to 0} \begin{matrix}\text{average velocity}\\ \text{between } a \text{ and } a+h\end{matrix}$$

Slope – Velocity Connection





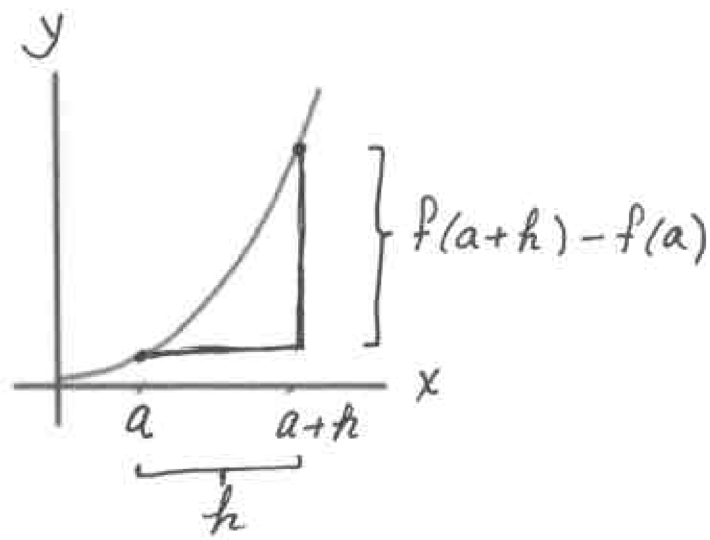
slope of secant = average velocity
PQ between Times a and a+h

slope of tangent = instantaneous velocity
at P at a

Slope Formula

The slope of the tangent line to $y = f(x)$ when $x = a$ is

$$m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$



Next – we need to learn about <u>limits</u>.
then how to calculate slopes & velocities
- these are e.g.'s of <u>derivatives</u>.