The Limit of a Function

Section 2.2

Informal Definition of Limit

L is the limit of $f(x)$ as x approaches a provided that we can make $f(x)$ as close to L as we may wish solely by making x close (but not equal) to a.

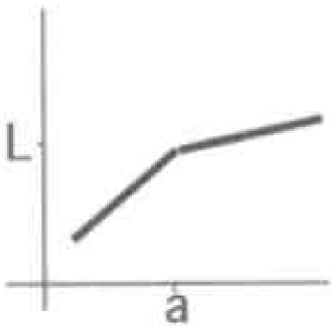
- As x approaches a, the value of $f(x)$ approaches L.
- Consequence: when x is near (but not equal to) a, $f(x)$ must be near L.

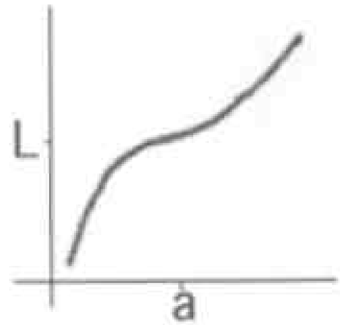
Notation: $\lim_{x \to a} f(x) = L$ on $f(x) \to L$ as $x \to a$

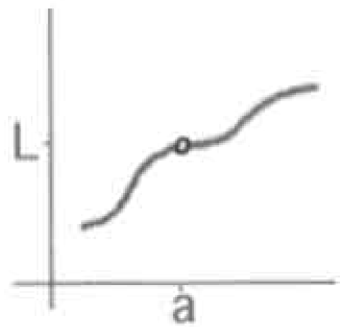
- $f(a)$ does not have to equal L; $f(a)$ does not even have to be defined.

Limit Pictures

Examples where $\lim_{x \to a} f(x) = L$:



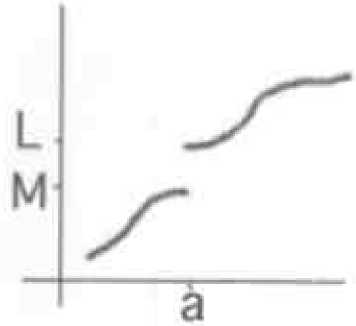




Remember – it does not matter whether $f(a)$ is defined.

More Limit Pictures

Example where $\lim_{x \to a} f(x)$ does not exist:



Notice that $\lim_{x \to a^-} f(x) = M$ and $\lim_{x \to a^+} f(x) = L$.

These <u>one-sided limits</u> exist.

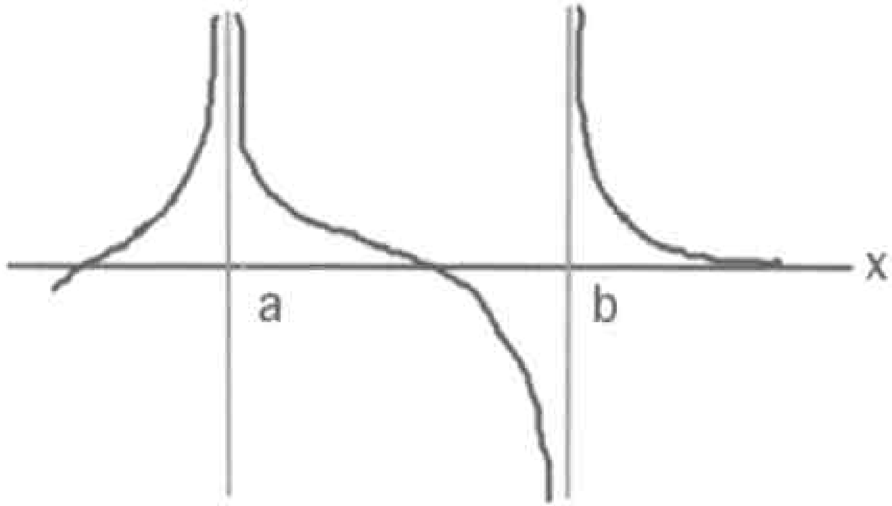
$\lim_{x \to a^+} f(x)$ is also called the <u>right-hand limit</u> at a.

Imp: $\lim_{x \to a} f(x) = L$ means both one-sided limits $= L$.

Vertical Asymptotes, Infinite Limits

If any of $\lim_{x \to a}$, $\lim_{x \to a^+}$, $\lim_{x \to a^-}$ is ∞ or $-\infty$, then the line x=a is a vertical asymptote.

Here, $x=a$ & $x=b$ are vertical asymptotes.



$\lim_{x \to a} f(x) = \infty$. $\lim_{x \to b^+} f(x) = \infty$ and $\lim_{x \to b^-} f(x) = -\infty$.

($\lim_{x \to b} f(x)$ does not exist.)

Examples - Calculating Infinite Limits

$$\lim_{x \to 0^+} \frac{2}{x(x+1)}$$

for x a bit larger than 0, denom. is + & near 0.

So ratio $2/x(x+1)$ is very big & +.

$$\lim_{x \to 0^+} 2/x(x+1) = \infty.$$

$$\lim_{x \to 0^-} \frac{2}{x(x+1)}$$

For x a bit < 0, denom. is − & near 0.

So $2/x(x+1)$ is very large & −.

$$\lim_{x \to 0^-} 2/x(x+1) = -\infty.$$

Note: $\lim_{x \to 0}$ does not exist since one-sided limits not equal.

Example - You try it

Show (as in the previous e.g.) that $\lim_{x \to 0} \frac{2}{x \sin x} = \infty$.
(Be sure to consider x's on either side of 0.)