Calculating Limits Using the Limit Laws

Section 2.3

Limit Laws

Assume $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Then

- $\lim_{x \to a}[f(x) \pm g(x)] = L \pm M$

  limit of $\frac{\text{sum}}{\text{diff}}$ = $\frac{\text{sum}}{\text{diff}}$ of limits

- $\lim_{x \to a} c f(x) = cM$ (for C constant)

- $\lim_{x \to a} f(x)g(x) = LM$

  limit of product = product of limits

- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ (provided $M \neq 0$)

  limit of quotient = quotient of limits

Useful Limit Facts

If r is a rational number,

$$\lim_{x \to a}[f(x)]^r = [\lim_{x \to a} f(x)]^r$$

can interchange limit & power ops.

If f is a polynomial,

$$\lim_{x \to a} f(x) = f(a).$$

can just evaluate at a

(more about this in section 2.5)

Limit Examples

$$\lim_{x\to 1}\underbrace{(x^2+1)^3}_{\text{polynomial}} = (1^2+1)^3 = 2^3 = 8$$

polynomial – so limit is value at 1.

$$\lim_{x\to 1}\sqrt{x^2+1} = \sqrt{\lim_{x\to 1} x^2+1} = \sqrt{2}$$

(½ power)

$$\lim_{x\to 1}\frac{x^2-1}{x-1} = \frac{0}{0}$$

"$\frac{0}{0}$" is undefined – so SIMPLIFY by factoring & canceling

$$= \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x\to 1}(x+1) = 2$$

Limit Examples – (cont.)

$$\lim_{x\to 0}\frac{\sqrt{1-x}-1}{x} = \, ?$$

"$\frac{0}{0}$" is undefined

SIMPLIFY by rationalizing numerator

$$= \lim_{x\to 0} \frac{(\sqrt{1-x}-1)}{x}\cdot\frac{(\sqrt{1-x}+1)}{(\sqrt{1-x}+1)}$$

$$= \lim_{x\to 0} \frac{1-x-1}{x(\sqrt{1-x}+1)} = \lim_{x\to 0} \frac{-\overset{1}{\cancel{x}}}{\cancel{x}(\sqrt{1-x}+1)}$$

(can cancel since $x \neq 0$)

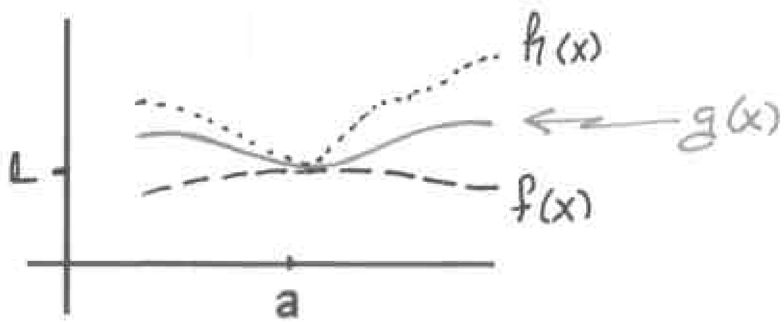
$$= \lim_{x\to 0} \frac{-1}{\sqrt{1-x}+1} = -\frac{1}{1+1} = -\frac{1}{2}$$

The Squeeze Law

If $f(x) \le g(x) \le h(x)$ for x near a (except possibly at a)

and if $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$,

then $\lim_{x\to a} g(x)$ must also $= L$.



Squeeze Law Example

Find $\lim_{x\to 0} x\sin(1/x)$ (note: $\sin(1/0)$ undefined)

Since $-1 \le \sin(1/x) \le 1$, $-|x| \le x\sin(1/x) \le |x|$

$$\lim_{x\to 0} |x| = 0 \quad \& \quad \lim_{x\to 0} -|x| = 0$$

These have the same limit, so the Squeeze Law implies:

$$\lim_{x\to 0} x\sin(1/x) = 0$$