Limits at Infinity; Horizontal Asymptotes

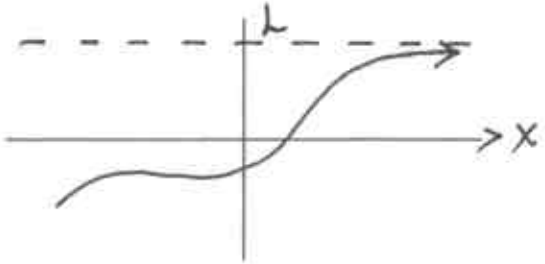
Section 2.6

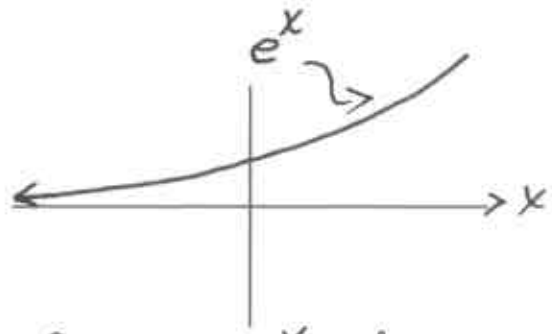
Horizontal Asymptotes

- can only occur as $x \to \infty$ or as $x \to -\infty$

- the horizontal line $y = L$ is a horizontal asymptote when

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$





e.g. $f(x) = e^x$ has horiz. asymptote $y = 0$ (as $x \to -\infty$)

Examples of Horizontal Asymptotes

- $\lim_{x\to\infty} \dfrac{x^3-7x+4}{2x^3+x+1} = \lim_{x\to\infty} \dfrac{1-7/x^2+4/x^3}{2+1/x^2+1/x^3} = 1/2$

divide top & bottom by highest power of x in denominator

*all Terms with x's on bottom $\to 0$

horiz. asymp. $y = 1/2$

- $\lim_{x\to-\infty} \dfrac{x^2+x-1}{5x^3-x^2} = \lim_{x\to-\infty} \dfrac{1/x+1/x^2-1/x^3}{5-1/x} = \dfrac{0}{5} = 0$

horiz. asymp. $y = 0$

* notice That $\lim_{x\to\infty} \frac{1}{x^n} = 0$ & $\lim_{x\to-\infty} \frac{1}{x^n} = 0$

for any positive integer n

More Examples – Limits at Infinity

- $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \dfrac{\sqrt{1+1/x^2}}{1+1/x} = \dfrac{\sqrt{1}}{1} = 1$

  "$\frac{\infty}{\infty}$" Divide Top & bottom by x $(x=\sqrt{x^2})$

- $\lim_{x\to\infty} \sqrt{x^2+x}-x = \lim_{x\to\infty} \dfrac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$

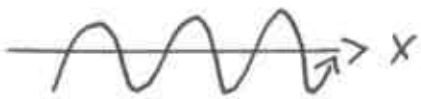
  "$\infty-\infty$" indeterminate

  $= \lim_{x\to\infty} \dfrac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \dfrac{1}{\sqrt{1+1/x^2}+1} = \dfrac{1}{2}$

  Divide Top & bottom by x

More Examples – Limits at Infinity

- $\lim_{x\to\infty} \sin x$ does not exist. As $x \to \infty$, $\sin x$ continues to oscillate between -1 & $+1$.



A limit must be single-valued.

$\lim_{x\to\infty}$ polynomial = $\lim_{x\to\infty}$ of highest order term
(also as $x \to -\infty$)

- $\lim_{x\to\infty} 5x^3 - 2x^2 + 1 = \lim_{x\to\infty} 5x^3 = \infty$
- $\lim_{x\to-\infty} 5x^3 - 2x^2 + 1 = \lim_{x\to-\infty} 5x^3 = -\infty$ (odd power of neg. #)
- $\lim_{x\to-\infty} 5x^6 - 2x^2 + 1 = \lim_{x\to-\infty} 5x^6 = \infty$ (even power)