# Tangents, Velocities, and Other Rates of Change

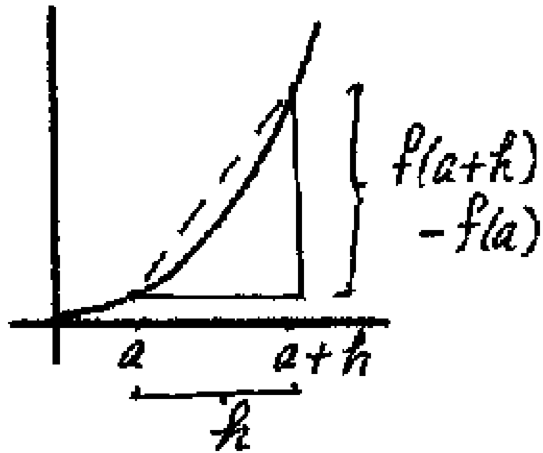
## Section 2.7

## Slope Formula

The slope of the tangent line to $y = f(x)$ when $x = a$ is

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(Review 2.1)




When f(x) represents the position of a moving object, we write v(a) instead of m. v(a) is the (instantaneous) velocity at time a.

## Slope, Velocity, Derivative

At the next lecture (section 2.8), we'll write $f'(a)$ instead of $m$.

$f'(a)$ is the derivative of $f$ at $a$.

$$\left.\begin{array}{l}\text{slope } m \\ \text{velocity } v(a) \\ \text{derivative } f'(a)\end{array}\right. = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

In section 2.7 you are expected to calculate $m$ by evaluating this limit.

## Example 1 – Calculating m From the Limit Definition

- Find the equation of the tangent line to $y = 4x^2 - 2x$ at (1, 2).

$a = 1$

$$\text{slope } m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{4(1+h)^2 - 2(1+h) - 2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + 8h + 4h^2 - \cancel{2} - 2h - \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{4h^2 + 6h}{h} = \lim_{h \to 0} 4h + 6 = 6$$

equation of Tangent line: $\dfrac{y-2}{x-1} = 6$ or $y = 6x - 4$

## Example 2 – Calculating m from the Limit Definition

- Find the slope of the tangent line to $y = \dfrac{4}{x-1}$ at $x = a$.

$$\text{slope } m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{a+h-1} - \frac{4}{a-1}}{h}$$

$$= \lim_{h \to 0} \frac{4(a-1) - 4(a+h-1)}{(a+h-1)(a-1)h}$$

$$= \lim_{h \to 0} \frac{-4\cancel{h}}{(a+h-1)(a-1)\cancel{h}} = \frac{-4}{(a-1)^2}$$

- Now find the slope for $a = -1$, $a = 2$, $a = 5$

Ans: $m(-1) = \frac{-4}{(-2)^2} = -1$, $m(2) = -4$, $m(5) = -1/4$

# Another Form for m

$$\boxed{m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

substitute $x = a+h$
then $h = x-a$
& $h \to 0$ becomes $x \to a$

OR

$$\boxed{m = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}}$$

$f'(a)$ can be called the rate of change of $f$ with respect to $x$ at $a$.

e.g. if $T(t)$ is temperature at time $t$, then $T'(2)$ is the rate of change of temperature with respect to time, at time 2.