Derivatives

Section 2.8

The Derivative

The derivative of f at a is

$$\boxed{f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$$

OR $$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$f'(a)$ is called the rate of change of f with respect to x at $x = a$.

Again, in this section, you are expected to calculate $f'(a)$ by evaluating the limit. (Hint: use the first formula)

Example – Derivative Calculation

Other than renaming m or $v(a)$ as a derivative $f'(a)$, there is nothing new in this section. So we'll just do one more (harder) derivative calculation.

- find $f'(a)$ when $f = \dfrac{1}{\sqrt{x}}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{a} - \sqrt{a+h}}{\sqrt{a+h}\,\sqrt{a}\,h}$$

mult top & bottom by $\sqrt{a} + \sqrt{a+h}$

$$= \lim_{h \to 0} \frac{\cancel{a} - (\cancel{a} + h)}{\sqrt{a+h}\,\sqrt{a}\,h(\sqrt{a} + \sqrt{a+h})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{a+h}\,\sqrt{a}(\sqrt{a} + \sqrt{a+h})} = \frac{-1}{\sqrt{a}\,\sqrt{a}\,2\sqrt{a}} = \frac{-1}{2a^{3/2}}$$