The Derivative as a Function

Section 2.9

The Derivative as a Function

Instead of calculating f' at $x = a$, we can leave the variable as x —

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ is called the derivative of f.

Note that it is a function of x, too.

- If $y = f(x)$, we have alternative notation:

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx} f(x)$$

You will need to recognize these other ways to write the derivative.

Differentiability Implies Continuity

Theorem: If f is differentiable at a, then f must be continuous at a.

Proof: Assume $f'(a)$ is finite.

Then $\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \left[\frac{f(x) - f(a)}{(x-a)}\right](x-a)$

$$= \left[\lim_{x \to a} \frac{f(x)-f(a)}{x-a}\right]\left[\lim_{x \to a} (x-a)\right]$$

$$= [f'(a)][0] = 0$$

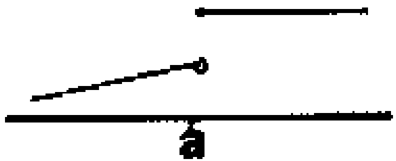
Notice that $\lim_{x \to a} [f(x) - f(a)] = 0$ is the same as

$\lim_{x \to a} f(x) = f(a)$, so f must be continuous at a.

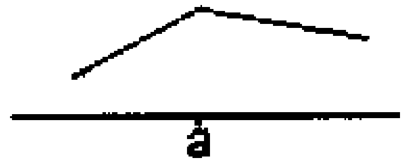
- We say "differentiability implies continuity."

Pictures

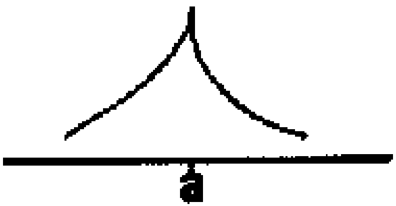
(at a)



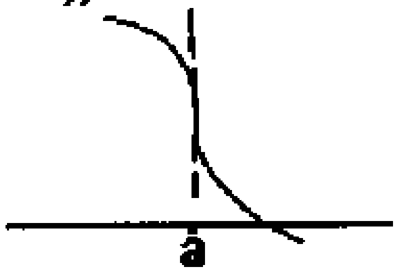
not continuous
& not differentiable



continuous but
not differentiable



continuous but
not differentiable
(infinite slope)



continuous but
not differentiable
(infinite slope)

continuous AND
differentiable

- $f'(a)$ exists means there is a (single-valued) finite slope at a.

Relating Graphs of f and f'

- Suppose we are given the graph of f, and are asked to sketch the graph of f':

For $x < A$ or $x > B$, f decreases & slope is < 0. i.e. $f' < 0$

At A & B slope is 0 i.e. $f' = 0$

For $A < x < B$, f increases & slope is > 0. i.e. $f' > 0$