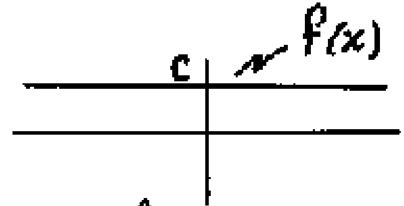
Derivatives of Polynomials and Exponential Functions

Section 3.1

Basic Differentiation Rules - 1

- If $f(x)=c$ then $f'(x)=0$.

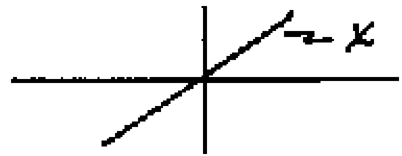


slope = 0

$$\frac{d}{dx}(c) = 0 \quad \text{or} \quad c' = 0$$

for c constant

- If $f(x)=x$ then $f'(x)=1$.



slope = 1

$$\frac{d}{dx}(x) = 1 \quad \text{or} \quad x' = 1$$

- <u>Power rule:</u> If $f(x) = x^n$ with n constant, then $f'(x) = nx^{n-1}$

i.e. $(x^n)' = nx^{n-1}$

Basic Differentiation Rules - 2

- Linearity of differentiation:

$$[a f(x) + b g(x)]' = a f'(x) + b g'(x)$$

when a & b are constants.

- Consequences:

• set $b=0$ & we get $[a f(x)]' = a f(x)$

• set $a=b=1$: $[f(x)+g(x)]' = f'(x) + g'(x)$

• set $a=1$, $b=-1$ $[f(x)-g(x)]' = f'(x) - g'(x)$

• note – $[a f(x) + b g(x) + c h(x)]'$

$$= [a f(x) + b g(x)]' + [c h(x)]'$$

$$= a f'(x) + b g'(x) + c h'(x)$$

etc. for >3 functions.
(for $a, b, c \ldots$ constants)

Example – Derivative of Polynomial

With linearity plus the power rule, it is easy to differentiate any polynomial.

- $[3x^5 - 12x^4 + 2x^2 + 5]'$

$$= 3(x^5)' - 12(x^4)' + 2(x^2)' + 5'$$

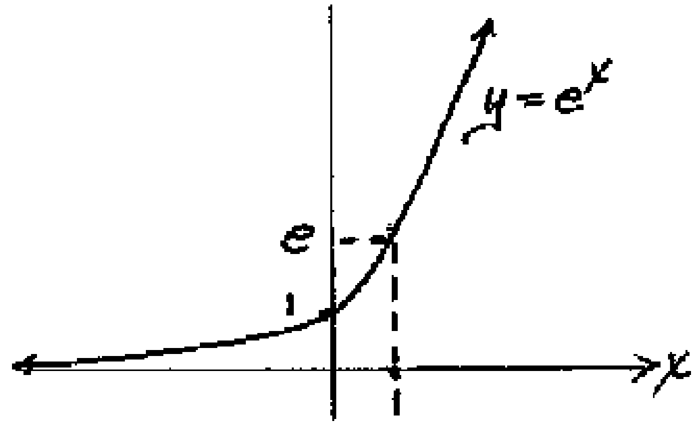
(linearity)

$$= 3(5x^4) - 12(4x^3) + 2(2x^1) + 0$$

$$= 15x^4 - 48x^3 + 4x$$

Derivative of Exponential Function

You should know the graph of e^x.



Fact: $\frac{d}{dx}(e^x) = e^x$

$\left(\text{or } (e^x)' = e^x\right)$

$e^0 = 1$

$\lim\limits_{x \to -\infty} e^x = 0$

$\lim\limits_{x \to \infty} e^x = \infty$

$e^x > 0$ for all x

Examples – Differentiation

- Find $f'(x)$ when $f(x) = 4x^4 - \frac{1}{x^2}$.

(Think of $\frac{1}{x^2}$ as x^{-2}, & use power rule.)

$$f'(x) = 4[x^4]' - 1[x^{-2}]'$$
$$= 4(4x^3) - 1(-2x^{-3}) = 16x^3 + 2x^{-3}$$

- Find $g'(t)$ when $g(t) = t\sqrt{t} + \frac{3}{t}$.

(Rewrite $g(t) = t^{3/2} + 3t^{-1}$)

$$g'(t) = \tfrac{3}{2}t^{1/2} + 3(-1t^{-2}) = \tfrac{3}{2}t^{1/2} - 3t^{-2}$$

- Find $f'(x)$ when $f(x) = 5e^x - 2$.

(Remember: $(e^x)' = e^x$ & $2' = 0$)

$$f'(x) = 5e^x$$