The Product and Quotient Rules

Section 3.2

Product Rule, Quotient Rule

The product rule:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

abbreviation: $\boxed{(uv)' = u'v + v'u}$

where u & v are functions of the same variable

e.g. $u = f(x)$ and $v = g(x)$

The quotient rule:

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

abbreviation: $\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$

Examples - Differentiation

- Differentiate $f(x) = \dfrac{x^2}{1+x^3}$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(2x)(1+x^3) - x^2(0+3x^2)}{(1+x^3)^2}$$

$$= \frac{2x + 2x^4 - 3x^4}{(1+x^3)^2} = \frac{2x - x^4}{(1+x^3)^2}$$

- Differentiate $g(y) = (y^2+1)(y^3+2y-3)$

$$(uv)' = u'v + uv'$$

$$g'(y) = 2y(y^3+2y-3) + (y^2+1)(3y^2+2)$$

$$= 2y^4 + 4y^2 - 6y + 3y^4 + 5y^2 + 2$$

$$= 5y^4 + 9y^2 - 6y + 2$$

Examples - Differentiation

- Differentiate $y = \frac{e^x}{x^2}$

could use quotient rule
OR could write as product
$y = e^x x^{-2}$

$$y' = (e^x)' x^{-2} + e^x (x^{-2})'$$
$$= e^x x^{-2} + e^x(-2x^{-3})$$
$$= \frac{e^x}{x^2} - \frac{2e^x}{x^3}$$

- Find $(fg)'(2)$ if $f(2) = 1$, $f'(2) = 2$, $g(2) = 3$, $g'(2) = 4$.

$$(fg)' = f'g + fg'$$
$$(fg)'(2) = f'(2)\,g(2) + f(2)\,g'(2)$$
$$= (2)(3) + (1)(4)$$
$$= 10$$

Example – Differentiation

- Find the slope of the tangent line to $y = \sqrt{x}e^{x}$ at $x=1$.

slope is $y'(1)$

$$y' = (\sqrt{x})' e^{x} + \sqrt{x}(e^{x})'$$

$$= \frac{1}{2}x^{-1/2}e^{x} + \sqrt{x}e^{x}$$

So $y'(1) = \frac{e}{2} + e = \frac{3}{2}e$