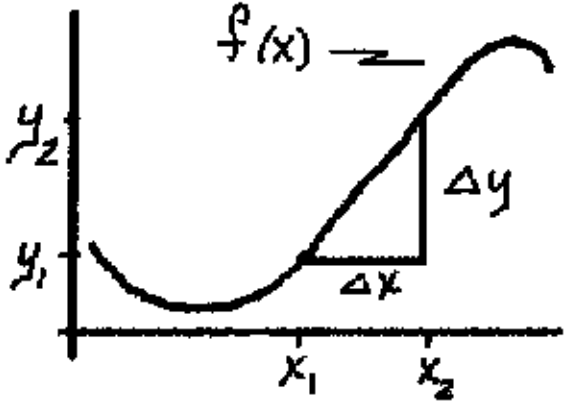
Rates of Change

Section 3.3

Δy / Δx Notation

Δy = change in y

Δx = change in x



$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$ (or $f'(x)$, or slope of Tangent)

So $\frac{dy}{dx}$ = <u>rate of change</u> of y with respect to x

units are y-units / x-units

Straight Line Motion

Let $s = f(t)$ be position of an object moving in a straight line.

- We know that $v(t) = ds/dt$ (or f'(t)) is its velocity –

  if s is in meters (m) & t in seconds (s), $\frac{ds}{dt}$ would be in m/s

- When is object at rest?

  when velocity = 0
  must solve v(t) = 0 for times at rest

- When is it moving forward (i.e. in positive direction)?

  when velocity > 0
  must solve inequality v(t) > 0

Example of Straight Line Motion

Given $s = t^3 - 9t^2 + 24t + 2$ m

Velocity: $v(t) = s' = 3t^2 - 18t + 24$ m/s

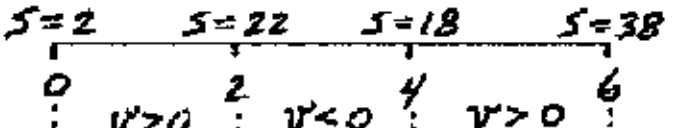
At rest when: $3t^2 - 18t + 24 = 0$

$3(t^2 - 6t + 8) = 0$

$3(t-2)(t-4) = 0$, $t = 2$ s or 4 s

Total distance (D) traveled during first 6 seconds:

must count forward & backward motion



D from $t=0$ to $t=2$ is $|s(2) - s(0)| = 20$ m

D " $t=2$ to $t=4$ " $|s(4) - s(2)| = 4$ m

D " $t=4$ to $t=6$ " $|s(6) - s(4)| = 20$ m

Total D = 44 m

Derivative Interpretations – 1

The problems in this section all deal with rates of change of physical quantities. Depending on your assignment, you may encounter:

- when $m = f(x)$ is mass of a thin straight rod to the left of position x, $\frac{dm}{dx}$ is called the linear density

- when $Q = f(t)$ represents electrical charge as a function of time, $\frac{dQ}{dt}$ is called the current

- when $n = f(t)$ is the population (# of animals or plank) at time t, $\frac{dn}{dt}$ is the growth rate

Derivative Interpretations – 2

- when $C = f(t)$ is the concentration of a substance in a chemical reaction, & t is time, then $\frac{dC}{dt}$ is called the rate of reaction

- when $C = f(x)$ is cost to produce x units (the cost function), then $\frac{dC}{dx}$ is called the marginal cost.
("marginal" in economics refers to a derivative)