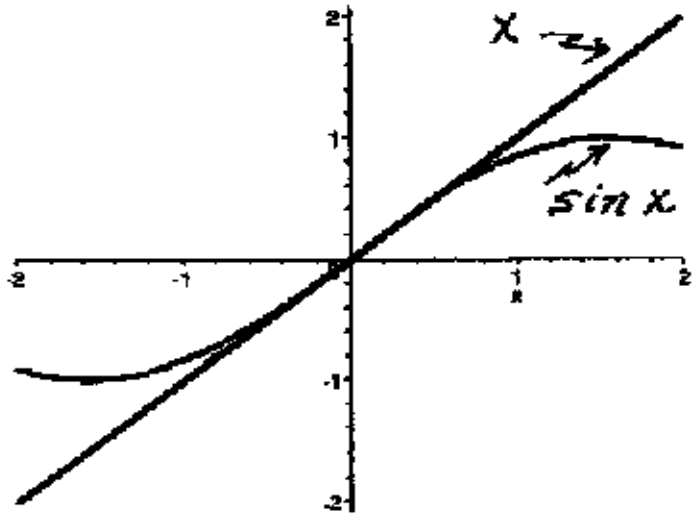
Derivatives of Trigonometric Functions

Section 3.4

An Important Trig Limit

- $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$



IMP: All angles in calculus should be in radians.

Examples – Trig Limits

- $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta} = \underset{\text{"}\frac{0}{0}\text{"}}{\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta}} \left(\dfrac{\cos\theta + 1}{\cos\theta + 1} \right)$

$$= \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} = \lim_{\theta \to 0} \frac{-\sin^2\theta}{\theta(\cos\theta + 1)}$$

(Remember: we know about $\sin\theta/\theta$)

$$= \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta} \right) \left(\frac{-\sin\theta}{\cos\theta + 1} \right)$$

$\to 1$ $\qquad$ $\to 0/2 = 0$

$$= (1)(0) = 0$$

The Derivative of Sin x

$$\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h - \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{(\sin x)(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} (\sin x)\left(\frac{\cos h - 1}{h}\right) + (\cos x)\left(\frac{\sin h}{h}\right)$$

$$= (\sin x)(0) + (\cos x)(1)$$

$$= \cos x$$

so: $\boxed{\frac{d}{dx}\sin x = \cos x}$

Examples – Trig Limits

- $\lim_{x\to 0} \dfrac{x}{\sin 4x} = \lim_{x\to 0} \left(\dfrac{4x}{\sin 4x}\right)\cdot\dfrac{1}{4} = 1\left(\dfrac{1}{4}\right) = \dfrac{1}{4}$

  as $x \to 0$, $4x \to 0$

- $\lim_{x\to 0} \dfrac{\tan x}{\sin 3x} = \lim_{x\to 0} \dfrac{\sin x}{(\cos x)(\sin 3x)}$

$$= \lim_{x\to 0} \sin x \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin 3x}$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{1}{\cos x}}_{\to 1} \cdot \underbrace{\frac{3x}{\sin 3x}}_{\to 1} \cdot \underbrace{\frac{x}{3x}}_{1/3} = \frac{1}{3}$$

Trig Derivative Formulas

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

MEMORIZE ! !

Examples - Trig Differentiation

- $\frac{d}{dx} x \sin x = (1)\sin x + x(\cos x)$

  $= \sin x + x\cos x$

- $\frac{d}{dx} \frac{\sin x}{\tan x + 1} = \frac{\cos x(\tan x + 1) - \sin x(\sec^2 x + 0)}{(\tan x + 1)^2}$

  $= \frac{\cos x \tan x + \cos x - \sin x \sec^2 x}{(\tan x + 1)^2}$