

Derivatives of Logarithmic Functions

Section 3.8

Derivative of $\ln x$

Wanted: y' when $y = \ln x$

\hookrightarrow means $e^y = x$

differentiate
implicitly:

$$e^y y' = 1$$
$$y' = \frac{1}{e^y} = \frac{1}{x}$$

So: $\frac{d}{dx} \ln x = \frac{1}{x}$

(note: $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$
for a positive constant a)

Differentiation Examples

$$\bullet \frac{d}{dx} \ln(x^2 + 2) = \frac{1}{x^2 + 2} (2x) = \frac{2x}{x^2 + 2}$$

$$\bullet \frac{d}{dx} \ln \sqrt{x} = \frac{1}{\sqrt{x}} \left(\frac{1}{2} x^{-1/2} \right) = \frac{1}{2x}$$

$$\underline{\underline{\text{OR}}} \quad \frac{d}{dx} \ln \sqrt{x^2} = \frac{d}{dx} \left(\frac{1}{2} \ln x \right) = \frac{1}{2x}$$

$$\bullet \frac{d}{dx} \ln \frac{x+1}{\sqrt{x^2+2}} = \frac{d}{dx} \left[\ln(x+1) - \frac{1}{2} \ln(x^2+2) \right]$$
$$= \frac{1}{x+1} - \frac{1}{2} \left(\frac{1}{x^2+2} \right) (2x) = \frac{1}{x+1} - \frac{x}{x^2+2}$$

simplify before differentiating

Logarithmic Differentiation

We know: $\frac{d}{dx} x^n = n x^{n-1}$

- variable base x -

$$\frac{d}{dx} a^x = a^x \ln a$$

- variable exponent x -

What if both base and exponent vary?

e.g. $\frac{d}{dx} x^x = ? ?$

For this we use The Technique called logarithmic differentiation.

→ see following example

Example 1 - Logarithmic Differentiation

- find $\frac{d}{dx} x^x$, for $x > 0$

$$\text{Let } y = x^x$$

we want y'

$$\ln y = x \ln x$$

ln each side

$$\frac{1}{y} y' = \ln x + x \left(\frac{1}{x}\right)$$

implicit differentiation

$$y' = y (\ln x + 1)$$

solve for y'

$$\text{Answer: } y' = x^x (\ln x + 1)$$

answer in terms of x

- log. differentiation is used for

$$\frac{d}{dx} [f(x)]^{g(x)} \quad \text{where base \& exponent vary.}$$

Example 2 – Logarithmic Differentiation

• find $\frac{d}{dx} \frac{(1+x^2)^{1/2}}{(1+x^4)^3}$

note: applying ln would simplify the function

let $y = \frac{(1+x^2)^{1/2}}{(1+x^4)^3}$

we want y'

$$\ln y = \frac{1}{2} \ln(1+x^2) - 3 \ln(1+x^4)$$

$$\frac{1}{y} y' = \frac{1}{2} \left(\frac{1}{1+x^2} \right) (2x) - 3 \left(\frac{1}{1+x^4} \right) (4x^3)$$

$$y' = y \left(\frac{x}{1+x^2} - \frac{12x^3}{1+x^4} \right)$$

$$= \frac{(1+x^2)^{1/2}}{(1+x^4)^3} \left(\frac{x}{1+x^2} - \frac{12x^3}{1+x^4} \right)$$

Answer in terms of x .