Antiderivatives

Section 4.10

Antiderivatives

- F is called an antiderivative of f if $F'(x) = f(x)$.

- We saw in section 4.2 (Mean Value Th.) that:

  any other antiderivative of f must look like $F(x) + C$, where C represents a constant.

- e.g. If $f(x) = x^2$, then $\frac{x^3}{3}$ is an antideriv. of x^2, and $F(x) = \frac{x^3}{3} + C$ is called the general antiderivative of x^2.

Examples of General Antiderivatives $F(x)$

$f(x)$	$F(x)$	$f(x)$	$F(x)$
x	$x^2/2 + C$	$\sin x$	$-\cos x + C$
x^n	$\dfrac{x^{n+1}}{n+1} + C$	$\dfrac{1}{1+x^2}$	$\tan^{-1} x + C$
e^x	$e^x + C$		

- If $g(x) = \ln|x|$, g is defined for all $x \neq 0$.

for $x > 0$, $g = \ln x$ & $g'(x) = 1/x$

for $x < 0$, $g = \ln(-x)$ & $g'(x) = \frac{1}{(-x)}(-1) = 1/x$ too.

The general antideriv. of $1/x$ is $\ln|x| + C$

More Antiderivative Examples

Find the general antiderivative $F(x)$ for the following funtions:

- $f(x) = x + 3e^x - 7\cos x$

$\frac{x^2}{2}$ e^x $\sin x$ antiderivs

$$F(x) = \frac{x^2}{2} + 3e^x - 7\sin x + C$$

Check That
$F'(x) = f(x)$

- $f(x) = \frac{2}{x} + \sqrt{x}$

$$F(x) = 2\ln|x| + \frac{x^{3/2}}{(3/2)} + C$$
$$= 2\ln|x| + \frac{2}{3}x^{3/2} + C$$

Check That
$F'(x) = f(x)$
remember $(\ln|x|)' = \frac{1}{x}$

More Examples

- find f if $f'(x) = 8x^3 + 2x$, and $f(1) = 5$

f is an antideriv. of f'

$f(x) = 8\frac{x^4}{4} + 2\frac{x^2}{2} + C$

$= 2x^4 + x^2 + C$

use extra condition $f(1) = 5$ to solve for C

$f(1) = 2 + 1 + C = 5$, so $C = 2$

answer: $f(x) = 2x^4 + x^2 + 2$

- find f if $f''(x) = 12x + 6$, $f(0) = 2$, and $f(1) = 3$

$f'(x) = 6x^2 + 6x + C$

$f(x) = 2x^3 + 3x^2 + Cx + D$

$f(0) = 2$ gives $f(0) = 0 + 0 + 0 + D = 2$, so $D = 2$

$f(1) = 3$ gives $f(1) = 2 + 3 + C + 2 = 3$, so $C = -4$

answer: $f(x) = 2x^3 + 3x^2 - 4x + 2$

More Examples

We are given the following data on a moving object. Find the position $s(t)$ at time t.

Remember: $v = s'$

$a = s''$

or

$a = v'$

- $v(t) = 3\sqrt{t} + 1 \text{ m}/\text{s}$, $s(4) = 25$ m

$s(t) = 2t^{3/2} + t + C$

$s(4) = 2(8) + 4 + C = 25$, so $C = 5$

answer: $s(t) = 2t^{3/2} + t + 5$ m • Check by differentiating.

- $a(t) = -32 \text{ ft}/\text{s}^2$, $v(0) = 12$ ft, $s(0) = 0$ ft

$v(t) = -32t + C$

$v(0) = 0 + C = 12$, so $C = 12$

$s(t) = -16t^2 + 12t + D$

$s(0) = 0 + 0 + D = 0$, $D = 0$

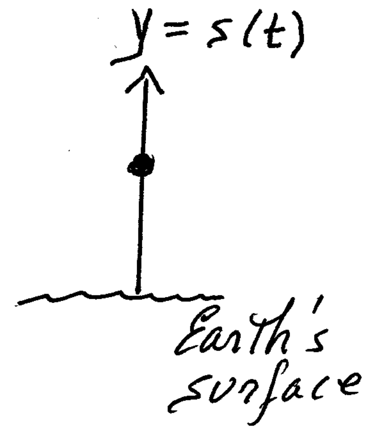
Answer: $s(t) = -16t^2 + 12t$ ft

Vertical Motion

- If we have an object rising or falling near Earth's surface, & if we assume that gravity is the only significant force acting on it, then the acceleration $a(t)$ is constant, and we use

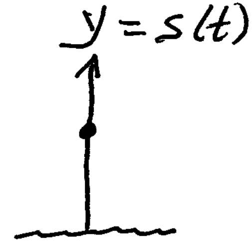
$$a(t) = -32 \text{ ft}/s^2 \quad \text{or} \quad a(t) = -9.8 \text{ m}/s^2 \qquad \text{(approx.)}$$

depending on our choice of units.



Example – Vertical Motion

- A ball is thrown straight upward from ground level with an initial speed of 160 ft/s. What is the maximum height that the ball attains? When does it fall back to ground?



given: $a(t) = -32, \quad v(0) = 160, \quad s(0) = 0$

then: $v(t) = -32t + C$

$v(0) = 0 + C = 160 \; ; \; C = 160$

$s(t) = -16t^2 + 160t + D$

$s(0) = 0 + 0 + D = 0 \; ; \; D = 0$

so $s(t) = -16t^2 + 160t$

Max. height occurs when $v(t) = 0$

$t = 160/32 = 5\,s.$

max height is $s(5) = 400\,ft.$

Falls back to ground when $s(t) = 0$; $\cancel{t = 0}$ or $10\,s.$

Answer