Indefinite Integrals and the Total Change Theorem

Section 5.4

Indefinite Integrals

The general antiderivative of f (see section 4.10) is called the indefinite integral of f —

$$\int f(x)\,dx = F(x) + C \quad \text{means} \quad F'(x) = f(x)$$

Don't confuse this with the definite integral of f — which is the number

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

e.g. $\int (x^3 + e^x)\,dx = \frac{x^4}{4} + e^x + C$

$$\int_0^1 (x^3 + e^x)\,dx = \left[\frac{x^4}{4} + e^x\right]_0^1 = \left(\frac{1}{4} + e\right) - \left(0 + e^0\right) = e - \frac{3}{4}$$

Do not forget the $+\,C$ in indefinite integrals.

Integral formulas

- Each differentiation formula we have learned has a corresponding integral formula:

e.g. $\frac{d}{dx} x = 1$ gives $\int dx = x + C$

$\frac{d}{dx} \sin x = \cos x$ " $\int \cos x \, dx = \sin x + C$

$\frac{d}{dx} \ln|x| = \frac{1}{x}$ " $\int \frac{dx}{x} = \ln|x| + C$

There is a table of integrals on p. 402 of your text – instead of memorizing it, you should just memorize the differentiation formulas and be able to deduce the integrals when you need them.

Total Change

If $f(x)$ ($= F'(x)$) represents the rate of change of F, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ is the (total) change in F between $x = a$ & $x = b$.

- e.g. If $s(t)$ represents the position of an object moving in a straight line, then

$v(t) = s'(t)$ is the rate of change of position

i.e. rate of displacement

displacement from $t=a$ to $t=b$ $= \int_a^b v(t)\,dt$

-> see next slide

Displacement & Distance Traveled

If $s(t)$ represents the position of an object moving in a straight line,

$v(t)$ is the rate of displacement and

displacement from $t = a$ to $t = b$ is $\int_a^b v(t)\,dt$

speed $= |v(t)|$ is the rate of change of distance traveled; so

distance traveled from $t=a$ to $t=b$ is $\int_a^b |v(t)|\,dt$

$a(t) = v'(t)$ is the rate of change of velocity

$$v(t) = \int a(t)\,dt$$

$$s(t) = \int v(t)\,dt$$

Example

$a(t) = 2t - 6$ is the acceleration of a moving object, with $v(0) = 5$. Find (a) the displacement for $0 \le t \le 8$, and (b) the distance traveled for $0 \le t \le 8$.

• get $v(t) = \int a(t)\,dt = \int (2t-6)\,dt = t^2 - 6t + C$

$$v(0) = 0 - 0 + C = 5, \text{ so } C = 5$$

$$v(t) = t^2 - 6t + 5$$

• (a) displacement $= \int_0^8 v(t)\,dt = \left[\frac{t^3}{3} - 3t^2 + 5t\right]_0^8$

$$= \left(\frac{8^3}{3} - 3(8)^2 + 40\right) - (0) = 56/3$$

• (b) dist. traveled $= \int_0^8 |v(t)|\,dt = \int_0^1 v(t)\,dt + \int_1^5 -v(t)\,dt$ ← NOTE

$$+ \int_5^8 v(t)\,dt = \frac{7}{3} + \frac{32}{3} + 27 = 40$$

$v>0$ 1 $v<0$ 5 $v>0$

$v = (t-1)(t-5)$