# The Substitution Rule

## Section 5.5

# Example 1 – Substitution in Indefinite Integrals

- $\int x\sqrt{x^2+1}\,dx \quad = \quad \int \frac{1}{2}\sqrt{u}\,du$

  > Idea: replace $\sqrt{x^2+1}$ by the simpler expression $\sqrt{u}$.

  $u = x^2+1$

  $du = 2x\,dx$

  $\left(\text{so } x\,dx = \frac{du}{2}\right)$

  > Rewrite $\int$ in terms of $u$ & $du$.

  $= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

  > Answer in terms of original variable.

  $= \frac{1}{3}(x^2+1)^{3/2} + C$

Check the answer by differentiating.

# Example 2 – Substitution in Indefinite Integrals

- $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \quad = \int 2e^u\,du$

Idea: replace $e^{\sqrt{x}}$ by the simpler expression $e^u$.

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

(so $\frac{dx}{\sqrt{x}} = 2\,du$)

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

Check answer by differentiating.

## Useful Comments on Substitution

- You want to choose a substitution $u = ?$ so that the entire integral can be rewritten in terms of $u$ and $du$.

- You want the $u$-integral to be easier than the original integral

  so $u = x$ never results in easier $\int$

- Answer in terms of the original variable.

# Examples 3 & 4 – Substitution in Indefinite Integrals

- $\int \frac{\sin x}{1+\cos^2 x}\,dx \quad = \int -\frac{1}{1+u^2}\,du = -\operatorname{Tan}^{-1} u + C$

  $u = \cos x \qquad\qquad = -\operatorname{Tan}^{-1}(\cos x) + C$

  $du = -\sin x\,dx$

- $\int \frac{dx}{x\ln x} \quad = \int \frac{du}{u} = \ln|u| + C$

  $u = \ln x \qquad = \ln|\ln x| + C$

  $du = \frac{dx}{x}$

# Example 1 – Substitution in Definite Integrals

- $\int_0^5 \frac{dx}{\sqrt{3x+1}} \quad = \quad \int_{?}^{?} \frac{1}{3} u^{-1/2} du$

$u = 3x + 1$

$du = 3\,dx$

? originally $x$ went from 0 to 5
new limits must give $u$-range

when $x = 0$, $u = 3(0)+1 = 1$
when $x = 5$, $u = 3(5)+1 = 16$

$$= \int_1^{16} \frac{1}{3} u^{-1/2} du$$

$$= \left[\frac{1}{3} \cdot 2u^{1/2}\right]_1^{16} = \frac{2}{3}(4) - \frac{2}{3}(1) = 2$$

# Example 2 – Substitution in Definite Integrals

- $$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \int_0^{\pi} \frac{1}{2}\cos u\,du$$

$u = x^2$ when $x = 0$, $u = 0$

$du = 2x\,dx$ " $x = \sqrt{\pi}$, $u = \pi$

$$= \left[\frac{1}{2}\sin u\right]_0^{\pi}$$

$$= \frac{1}{2}(0) - \frac{1}{2}(0) = 0$$

# Example 3 – Substitution in Definite Integrals

- $\int_1^2 x\sqrt{x-1}\,dx$

$u = x-1$ , so $x = u+1$ when $x=1$, $u=0$

$du = dx$ 〃 $x=2$, $u=1$

$$= \int_0^1 (u+1)\sqrt{u}\,du$$

$$= \int_0^1 (u^{3/2} + u^{1/2})\,du = \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1$$

$$= \left(\frac{2}{5} + \frac{2}{3}\right) - (0) = 16/15$$