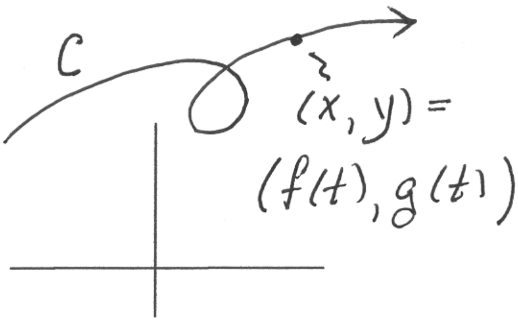
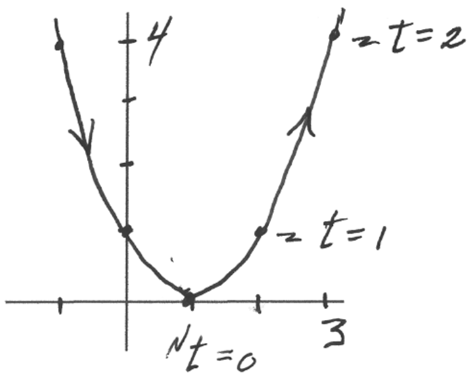
Parametric Curves



- describes object moving in plane
- represented by parametric curve C given by parametric equations

$$x = f(t),\ y = g(t)$$

e. g. $\quad x = t+1,\ y = t^2$



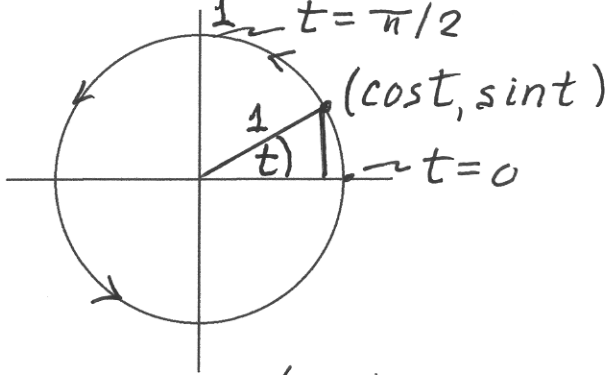
t	x	y
-2	-1	4
-1	0	1
0	1	0
1	2	1
2	3	4

arrows point in direction of increasing t

(useful to think of t as time)

Parametric Circles

- The unit circle is given by $x = \cos t,\ y = \sin t$ for $0 \le t \le 2\pi$



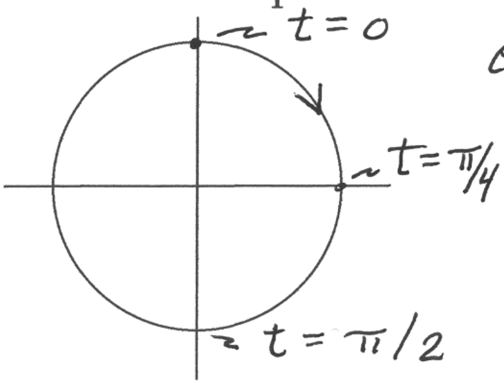
note: $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$ shows that the points are on ○

- start at $(1,0)$ for $t=0$
- at $(0,1)$ when $t = \pi/2$
- goes once around, <u>counter-clockwise</u>

- circle of radius a:

  $x = a\cos t,\ y = a\sin t$

- Different parametric equations can represent the same curve:



e.g. $x = \sin 2t,\ y = \cos 2t,\ 0 \le t \le \pi$

- $x^2 + y^2 = 1$
- start at $(0,1)$ for $t=0$
- goes once around ○, <u>clockwise</u>

Eliminating Parameters

Sometimes we can eliminate the parameter of a parametric curve to get an equation in x and y.

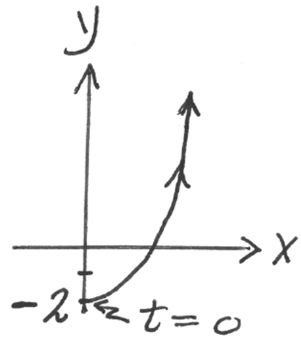
- e. g. $x = 5\cos t, \quad y = 2\sin t$

  since we know cost & sint are related by $\cos^2 t + \sin^2 t = 1$, we get $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$, which we should recognize as an ellipse.

- e. g. eliminate the parameter & sketch: $x = \sqrt{t}, \quad y = 3t - 2$

  $t = x^2$, so $y = 3x^2 - 2$

  since $x = \sqrt{t}$ must be ≥ 0, this curve is half a parabola.



Importance of Parametric Curves

- can represent complicated paths that cannot be given by functions

- used in computer-aided design (CAD) systems (called Bézier curves)

- used for Post-script fonts

- most paths in 3 dimensions are described parametrically

$$x = f(t), \quad y = g(t), \quad z = h(t)$$