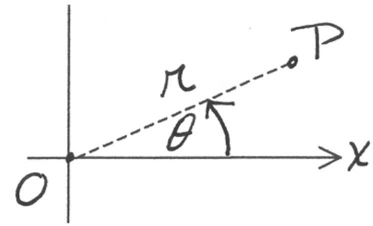
Definition of Polar Coordinates

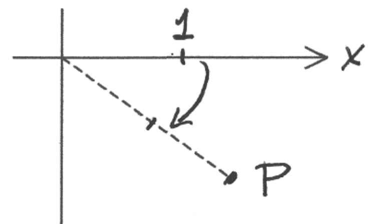


- A point P has polar coordinates (r, θ) with $r > 0$ and $\theta > 0$ if:

  θ is the angle measured counterclockwise from the positive x-axis to OP,

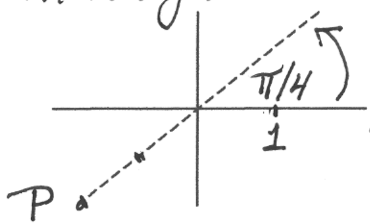
  and P is distance r from the origin.

- If $\theta < 0$, it is measured clockwise



$P = (2, -\pi/4)$

- If $r < 0$, the terminal line of θ is reflected through the origin.



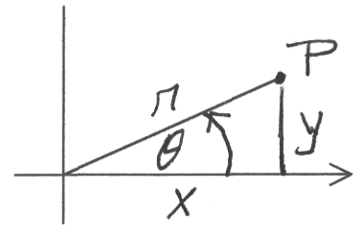
$P = (-2, \pi/4)$ or $(2, 5\pi/4)$ or $(2, -3\pi/4)$ or ...

Conversion Between Polar and Cartesian Coordinates

- We just saw that a point can have infinitely many polar coords.
- Conversion relations:

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$x^2 + y^2 = r^2 \qquad \tan\theta = y/x$$



e. g. convert to polar cords (r, θ): $x^2 - y^2 = 1$

answer: $r^2\cos^2\theta - r^2\sin^2\theta = 1$

e. g. convert to Cartesian cords (x, y): $r = 2 - \sin\theta$

(make use of $y = \underbrace{r\sin\theta}$) $\qquad r^2 = 2r - \underbrace{r\sin\theta}$

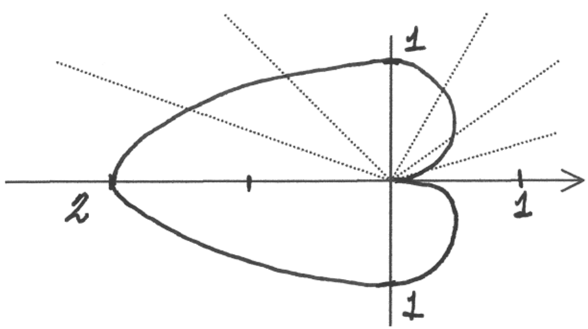
answer: $x^2 + y^2 = 2\sqrt{x^2+y^2} - y$

Example 1 – Sketching Polar Curves

Sketch the graph of $r = 1 - \cos\theta$.

- Always do θ–interval of length at least 2π.
- Here, since $\cos\theta$ has period 2π, the graph repeats if we go past $0 \le \theta \le 2\pi$.

θ	$0 \to \pi/2 \to \pi \to 3\pi/2 \to 2\pi \to$
r	$0 \nearrow 1 \nearrow 2 \to 1 \to 0$ graph repeats

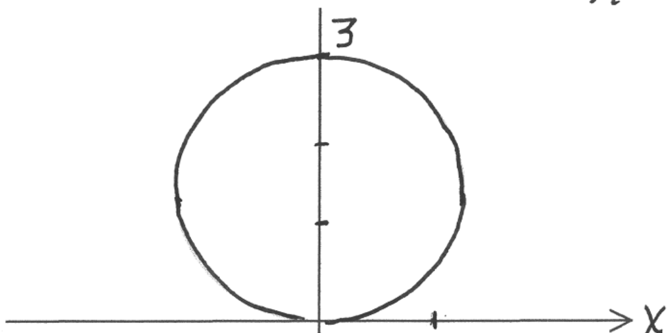


- heart shaped – called a cardiod
- $r = a\left(1 \pm \begin{matrix}\cos\theta \\ \sin\theta\end{matrix}\right)$ are all cardiods. a can be + or –

Example 2 – Sketching Polar Curves

Sketch the graph of $r = 3\sin\theta$.
(As in the last e. g., $\sin\theta$ has period 2π, so the graph repeats if we go past $0 \le \theta \le 2\pi$.)

$$\begin{array}{l}
\theta \quad 0 \to \pi/2 \to \pi \to 3\pi/2 \to 2\pi \\
\hline
r \nearrow 0 \nearrow 3 \searrow \underbrace{0 \searrow -3 \nearrow 0}_{r<0}
\end{array}$$

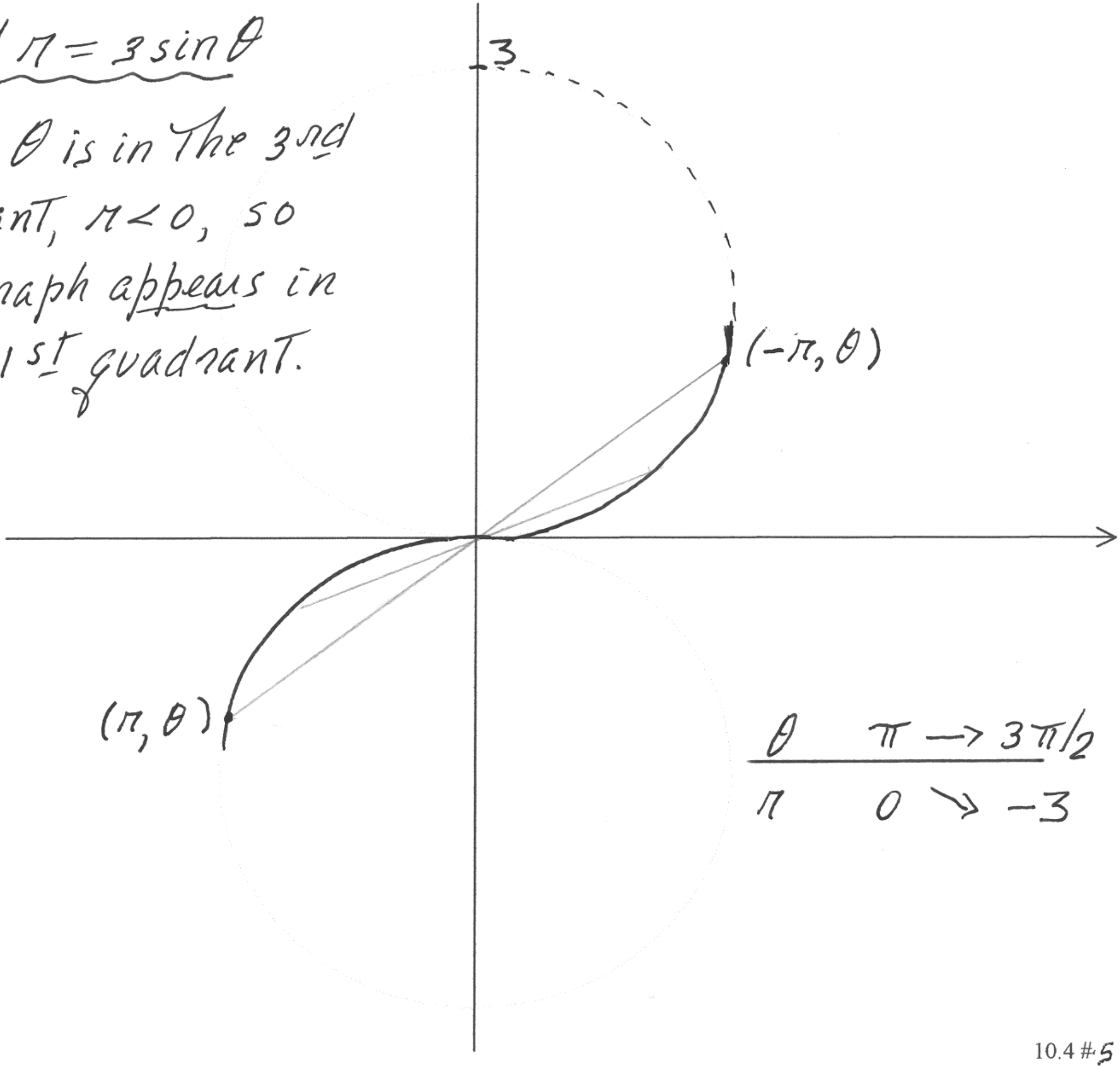


this turns out to be a circle!

- As θ goes from 0 to π, this circle is traced once 1
- As θ goes from π to 2π, $r<0$, & we retrace the circle.

Part of $r = 3\sin\theta$

when θ is in the 3^{rd} quadrant, $r < 0$, so the graph appears in the 1^{st} quadrant.



θ	$\pi \rightarrow 3\pi/2$
r	$0 \searrow -3$

Example 3 – Sketching Polar Curves

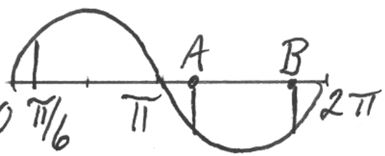
Sketch the graph of $r = 1 + 2\sin\theta$. (Period 2π)

θ	$0 \to \pi/2 \to \pi \to 3\pi/2 \to 2\pi \to$	
r	$1 \nearrow 3 \to 1 \to \underbrace{-1 \nearrow}_{r<0} 1$	graph repeats

• find where $r < 0$. $r = 0$ when $\sin\theta = -1/2$.

know $\sin \pi/6 = +1/2$

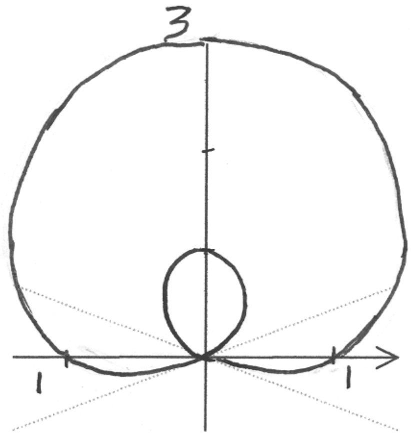
$\sin\theta$ →



$A = \pi + \pi/6 \qquad B = 2\pi - \pi/6$

$r < 0$ between A & B.

A limaçon



Tangents to Polar Curves

- The slope of a tangent line is $\dfrac{dy}{dx}$.

- If $r = f(\theta)$ is a polar curve, we use

$$\left.\begin{aligned} x &= r\cos\theta & \quad \& \quad y &= r\sin\theta \\ &= f(\theta)\cos\theta & &= f(\theta)\sin\theta \end{aligned}\right\} \text{to get parametric equations for } r = f(\theta)$$

- e. g. find $\dfrac{dy}{dx}$ for the cardiod $r = 1 - \cos\theta$.

$$x = r\cos\theta = \cos\theta - \cos^2\theta, \quad y = r\sin\theta = \sin\theta - \sin\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{(dy/d\theta)}{(dx/d\theta)} = \frac{\cos\theta - \cos^2\theta + \sin^2\theta}{-\sin\theta + 2\cos\theta\sin\theta}$$

When $\theta = \pi/4$, slope $= \dfrac{\sqrt{2}/2 - 2/4 + 2/4}{-\sqrt{2}/2 + 2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}} = \dfrac{\sqrt{2}}{2-\sqrt{2}}$.