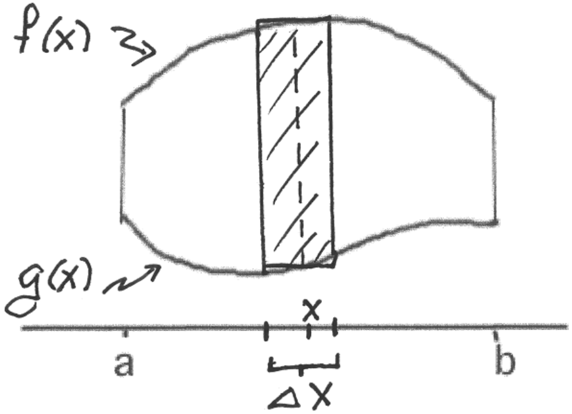
Area Between Two Curves



- We want the area A of the region between f and g for $a \le x \le b$.
- Slice into vertical slices, of width Δx.

- Area of slice ≈ area of of height $f(x) - g(x)$
- Add up rectangle areas (get Riemann sum) (REVIEW: section 5.2)

$$\text{Area} \approx \sum [f(x) - g(x)] \Delta x$$

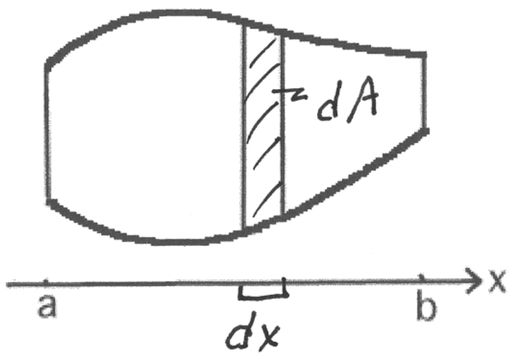
- Let $\Delta x \to 0$ (so more, thinner, slices)

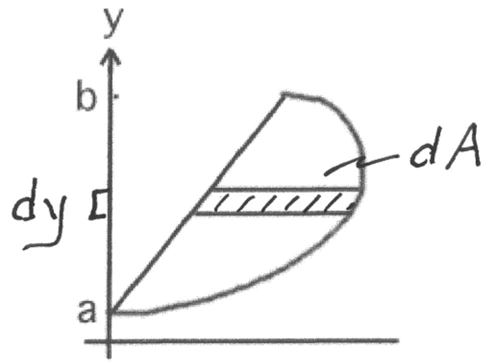
the sum Σ turns into $\int$ & Δx becomes dx

$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx$$

Setting up Area Integrals

- Draw a picture of the region & of a "slice" dA of area.





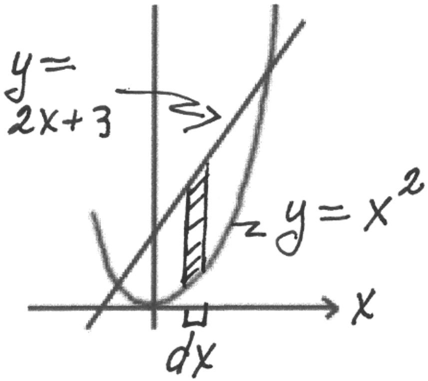
- $dA = (\text{height})\, dx$
  express height in terms of x
- $\text{Area} = \int_{x=a}^{x=b} dA$

$dA = (\text{width})\, dy$
express width in terms of y

$\text{Area} = \int_{y=a}^{y=b} dA$

Area Example 1

Find the area of the region bounded by $y = x^2$ and $y = 2x + 3$.



To get a & b: find x-value of intersection of curves:

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0, \quad (x+1)(x-3) = 0$$

$$x = -1 \ \& \ x = 3$$

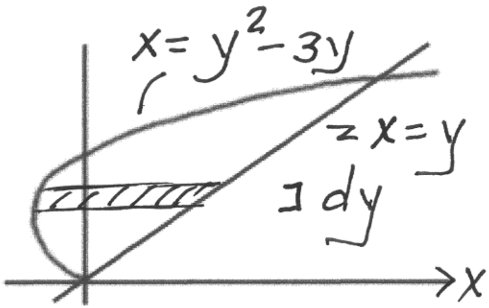
$$dA = (\text{height})\,dx = (2x + 3 - x^2)\,dx$$

$$\text{Area} = \int_{-1}^{3} (2x + 3 - x^2)\,dx = \left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^{3}$$

$$= 9 - -5/3 = 10\ 2/3$$

Area Example 2

Find the area of the region bounded by $x = y$ and $x = y^2 - 3y$.



Intersect when

$$y = y^2 - 3y$$

$$y^2 - 4y = 0, \quad y(y-4) = 0$$

$$y = 0 \text{ \& } y = 4$$

$$dA = (\text{width})\, dy$$

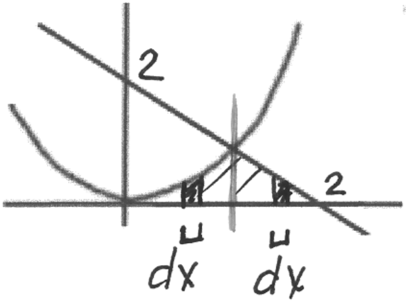
$$= [y - (y^2 - 3y)]\, dy = [4y - y^2]\, dy$$

$$\text{Area} = \int_0^4 [4y - y^2]\, dy = \left[2y^2 - y^3/3\right]_0^4$$

$$= 32 - 64/3 = 10\ 2/3$$

Area Example 3

Find the area of the region bounded by $y = x^2$, $y = 2 - x$ and $y = 0$.



- Intersect when $x^2 = 2 - x$

  $x^2 + x - 2 = 0$, $(x+2)(x-1) = 0$

  $x = 1$ & $x = -2$

  this is the value we want

- Break into 2 areas –

  A_1 for $0 \le x \le 1$, A_2 for $1 \le x \le 2$

$dA_1 = (\text{height})\,dx$
$= x^2\,dx$

$dA_2 = (\text{height})\,dx$
$= (2-x)\,dx$

$$\text{Area} = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx = \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$= 5/6$