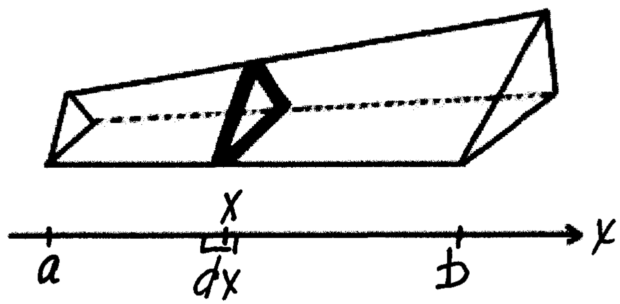
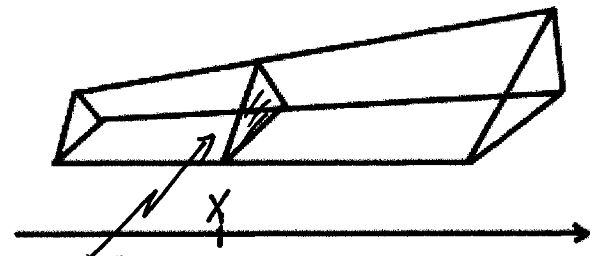
Volumes by Cross-sections

- Wanted: volume V of an irregular solid




Vol. slice ≈ (X-sectional area)(Thickness)

- $\boxed{dV = A(x)\,dx}$

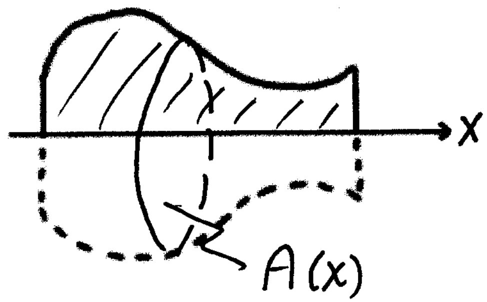
$$V = \int_{x=a}^{x=b} dV = \int_a^b A(x)\,dx$$

- Useful when you have a formula for X-sectional area $A(x)$.

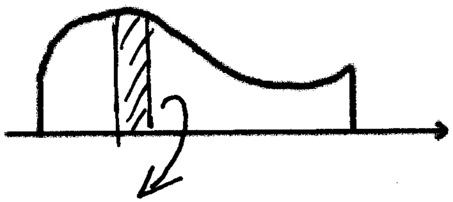
## Volume of Solid of Revolution - by Disks

- Solid is generated by revolving region between curve & axis about axis

- X-sectional $A(x)$ is circular disk




$A(x) = \pi (\text{radius})^2$



slice revolves to dV of

- $dV = \pi (\text{radius})^2$ thickness

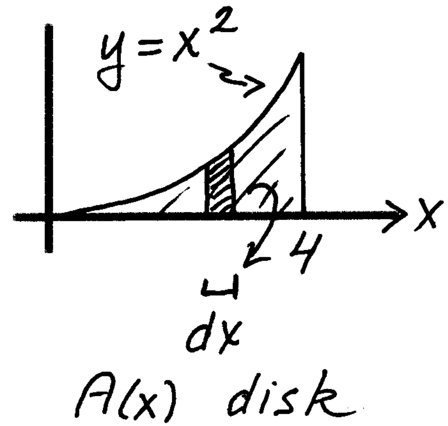
- This setup used for variety of axes of revolution — x- or y-axis, or lines x or y = constant.

- dV must be expressed in terms of thickness variable

Example 1 – Volume by Disks

Find the volume of the solid generated by revolving the region between $y = x^2$ and the $x$-axis, for $0 \le x \le 4$, about the $x$-axis.

- draw 2-D picture




- $dV = \pi (\text{radius})^2 dx$

radius $= x^2$

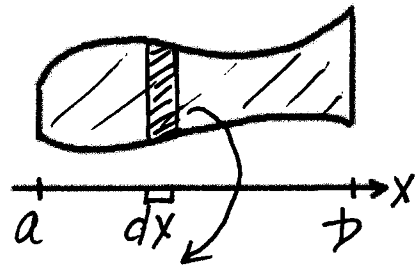
so $dV = \pi x^4 dx$

- $V = \int_0^4 \pi x^4 dx = \pi \left[ \frac{x^5}{5} \right]_0^4$

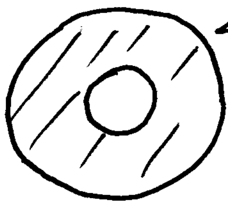
$= \frac{1024}{5} \pi$

# Volume of Solid of Revolution – by Washers

- Solid is generated by revolving region between 2 curves about non-intersecting axis




- X-sectional $A(x)$ is washer



outer radius $r_{out}$

inner radius $r_{in}$

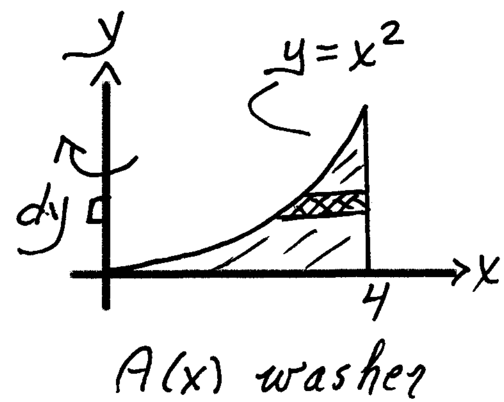
- $A(x) = \pi (r_{out})^2 - \pi (r_{in})^2$

$dV = A(x) \underbrace{dx}_{\text{thickness}}$

- axis of revolution might be any horiz. or vertical line.

Example 2 – Volume by Washers

Find the volume of the solid generated by revolving the region between $y = x^2$ and the $x$-axis, for $0 \le x \le 4$, about the $y$-axis.




- draw 2-D picture
- $dV = [\pi (r_{out})^2 - \pi (r_{in})^2] dy$

  $r_{out}$ is 4

  $r_{in}$ is the x-value on $y = x^2$ expressed in terms of thickness variable y.

  so $r_{in} = \sqrt{y}$.

- So $dV = (16\pi - \pi y) dy$, y goes from 0 to $4^2$

  $V = \pi \int_0^{16} (16 - y) dy = 128\pi$