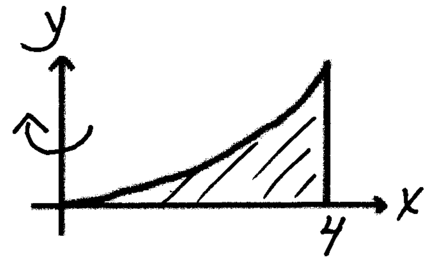
Volumes by Cylindrical Shells

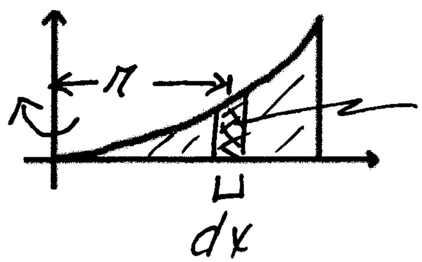
Section 6.3

Volume by Cylindrical Shells - 1

Find the volume of the solid generated by revolving the region between $y = x^2$ and the x-axis, for $0 \le x \le 4$, about the y-axis.



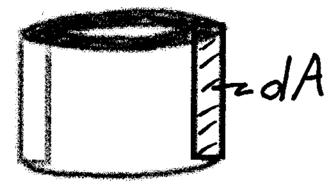
- If we slice horizontally, we get washer-shaped X-sections
  - see 6.2 example 2
- Instead, we'll slice vertically — so look at a vertical slice of area, & imagine revolving it about the y-axis.



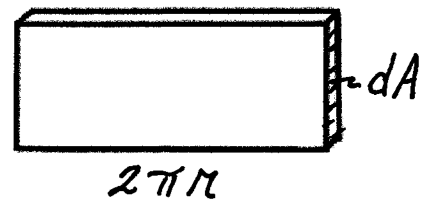
$dA = (\text{height})\, dx$

Volume by Cylindrical Shells – 2

dA revolves to a <u>cylindrical shell</u>, of circumference $2\pi r$ with walls of thickness dx.



If we slit the shell vertically, & flatten it, we see that its volume is $\approx 2\pi r\, dA$



This gives us the differential formula

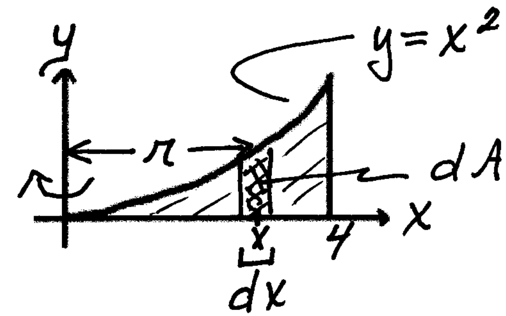
$$dV_{ol} = 2\pi r_{adius}\, dA_{rea},$$

or just $\boxed{dV = 2\pi r\, dA}$ & $\boxed{V = \int dV}$

(r & dA must be written in terms of the thickness variable)

Volume by Cylindrical Shells – 3

Working out our sample problem:



dV = cyl. shell

$= 2\pi r \, dA$

where $dA = (\text{height})\,dx$

$= x^2 \, dx$

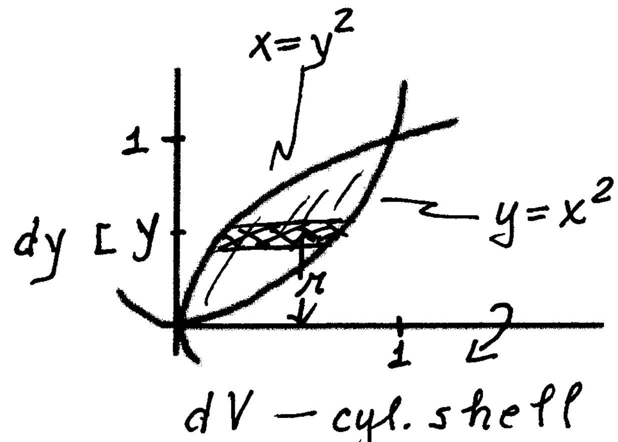
& r (in terms of x) $= x$

So $dV = 2\pi x^3 dx$ & $V = \int_0^4 2\pi x^3 dx = 2\pi \left[\frac{x^4}{4} \right]_0^4$

$= 128\pi$

Example 1 – Cylindrical Shells

Revolve the region between $y = x^2$ and $x = y^2$ for $0 \le x \le 1$ about the x-axis. Find the volume.



- need 2-D picture

- $dV = 2\pi r\, dA$

where

$dA = (\text{width})\,dy$ .... in Terms of y

$= (x_{right} - x_{left})\,dy$ "

$= (\sqrt{y} - y^2)\,dy$ & $r = y$

So $dV = 2\pi(y^{3/2} - y^3)\,dy$

- $V = 2\pi \int_0^1 (y^{3/2} - y^3)\,dy = 2\pi\left[\frac{2}{5}y^{5/2} - \frac{1}{4}y^4\right] = 0.3\pi$

Example 2 – Cylindrical Shells

Revolve the region between $y = x^2$ and $y = x$ for $0 \le x \le 1$ about the line $x = -1$. Find the volume.

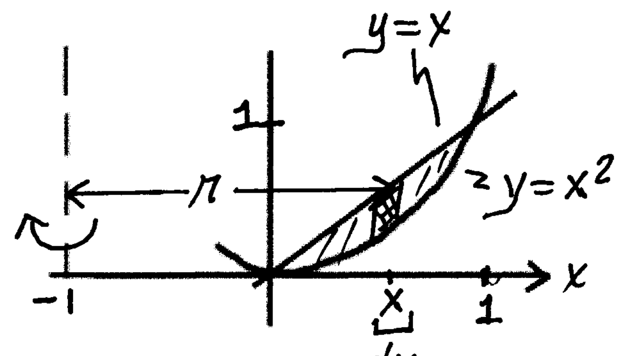
- need 2-D picture
- $dV = 2\pi r \, dA$

  & $dA = (\text{height})\, dx$

  $= (x - x^2)\, dx$

  & $r = x + 1$

  $dV = 2\pi (x+1)(x-x^2)\, dx$



dV – cyl shell

- $V = 2\pi \int_0^1 (\cancel{x^2} - x^3 + x - \cancel{x^2})\, dx = 2\pi \left[-\frac{x^4}{4} + \frac{x^2}{2} \right]_0^1$

  $= 2\pi \left[-\frac{1}{4} + \frac{1}{2} \right] = \frac{\pi}{2}.$