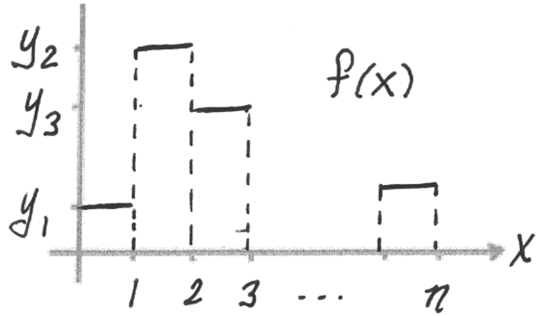
Average Value of $f(x)$

The average value of $f(x)$ on the interval $[a,b]$ is defined to be

$$f_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

- For an f with graph $\longrightarrow$



you can see that

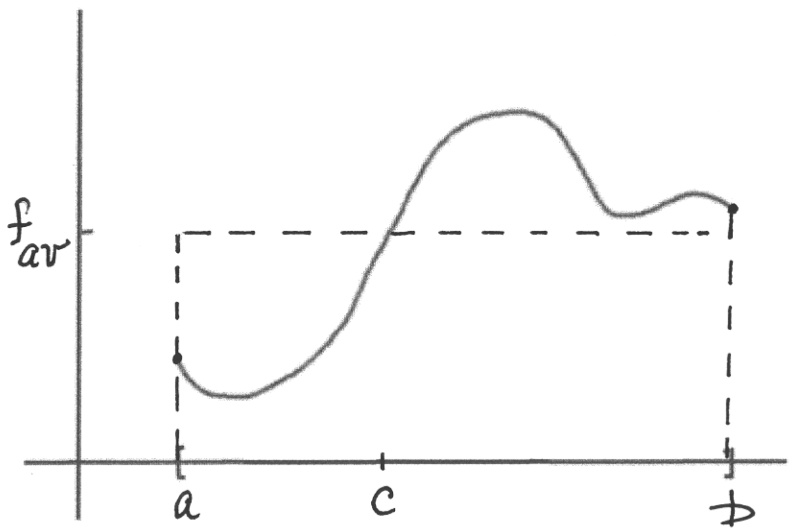
$$\int_0^n f(x)\,dx = \text{Area under } f$$
$$= y_1 + y_2 + \cdots + y_n$$

So $\frac{1}{n-0}\int_0^n f(x)\,dx = (y_1 + y_2 + \cdots + y_n)/n$, which is our usual concept of the average of n discrete values.

Mean Value Theorem for Integrals

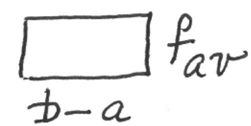
Theorem: When f is continuous on $[a,b]$, there is a number c in $[a,b]$ that satisfies $f(c) = f_{av}$.

consequence : $\int_a^b f(x)\,dx = f(c)(b-a)$



picture when $f \geq 0$

The area of the rectangle



equals $\int_a^b f(x)\,dx$

Average Value Example

For $f(x)=x^2-2x+1$ on $[0,3]$

(a) find f_{av}

(b) find c such that $f(c)=f_{av}$

(c) sketch f & a rectangle whose area equals the area under f.

(a) $f_{av} = \frac{1}{3-0}\int_0^3 (x^2-2x+1)\,dx = \frac{1}{3}\left[\frac{x^3}{3}-x^2+x\right]_0^3$

$= \frac{1}{3}[3] = 1$

(b) solve $c^2-2c+1=1$

$c(c-2)=0$

$c=0$ or 2

(c) picture on right