Frequently Asked Questions on Volume Integrals

(Sections 6.2 and 6.3)

1 Why is my volume answer twice as large as it should be?

2 Why is my volume answer negative? or zero?

3 Some of my cross–sections look like disks and some like washers. Which formula do I use?

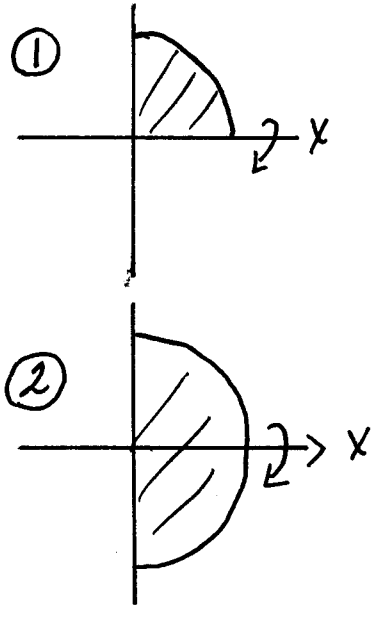
4 How do I choose between disks (or washers) and shells?

5 How do I choose my limits of integration?

6 I have a problem where the X–sections aren't disks or washers. How do I start ?

Why is my volume answer twice as large as it should be?

Answer: The area that you revolve to generate a solid should be no larger than necessary to generate the solid.



When revolved about the x-axis, regions ① & ② generate the same solid.

However if you set up your volume integral using (all of) region ②, your answer will be double the correct answer.

Why is my volume answer negative? or zero?

Answer: If you were using washer X–sections, likely you mixed up the inner and outer radii, or else you used the wrong limits of integration.

If you are using shells, you must make sure that your radius is positive and also that the length (or width) of your area elements is positive.

The above errors result in a negative dV.

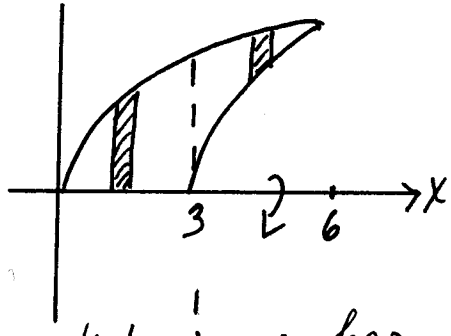
You need a good picture to show the radii and area elements.

If you got a zero answer, you likely used twice as much area as was necessary to generate the solid, and also made one of the above errors for part of the area.

Some of my cross–sections look like disks and some like washers. Which formula do I use?

Answer: Divide the solid into a sum — one part where the X–sections are disks and the other with washer X–sections.

e.g.



$V = \int x \text{ from } 0 \text{ to } 3$

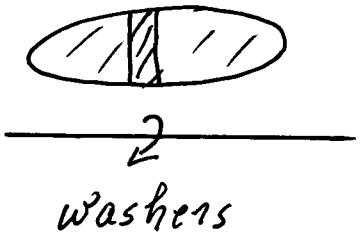
$+ \int x \text{ from } 3 \text{ to } 6$

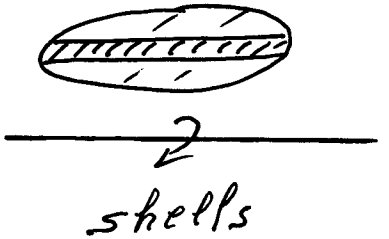
How do I choose between disks (or washers) and shells?

Answer: Generally you choose the method that is easier to set up. However, if the method you choose leads to a difficult integral, try the other method.

Remember: you have disks or washers when you slice perpendicular to the axis of revolution,

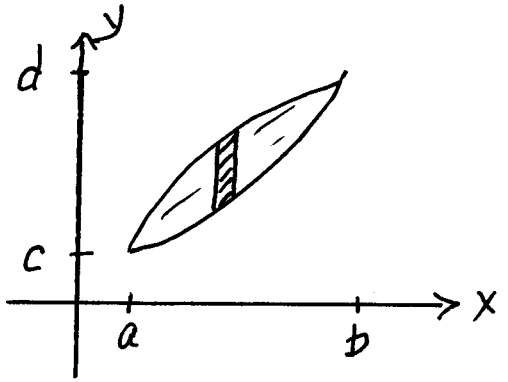
and you have shells when you slice your region parallel to the axis of revolution.





How do I choose my limits of integration?

Answer: You want your integral to incorporate all the area slices necessary to generate the solid .

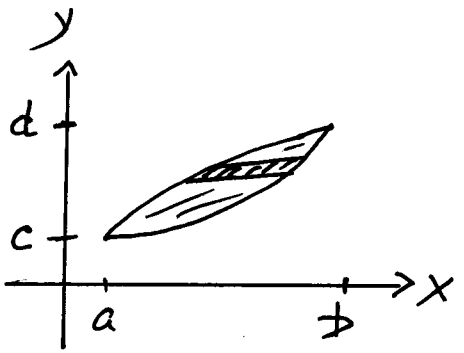


Variable of integration is x;

x goes from a to b.

(vertical slices start at x=a,

and end at x=b)



Variable of integration is y;

y goes from c to d.

(horizontal slices start at y=c,

and end at y=d)

I have a problem where the X–sections aren't disks or washers. How do I start?

Answer: It appears that you don't even have a solid of revolution. Back on the first slide for Section 6.2,we got the formula

$$V = \int_a^b A(x)\,dx$$

$A(x)$: X-sectional area

dx: thickness of slice

If A(x) is a simple geometric shape — e.g. square, rectangular, triangular, etc. — you should be able to use elementary geometry to get a formula for A(x).