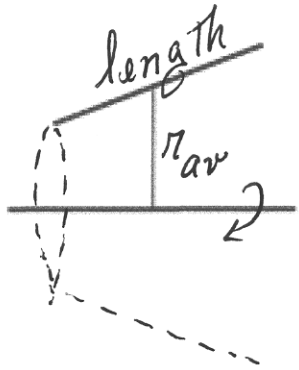
# Area of Surface of Revolution

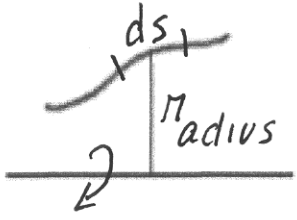



If we revolve a line segment about an axis (see picture), we obtain a frustrum of a cone.

Its surface area is

$2\pi r_{\text{average}}$ length of segment

We want the surface area $S$ obtained when we revolve a smooth curve about an axis —




circumference $= 2\pi r$

$$dS = 2\pi r\, ds$$

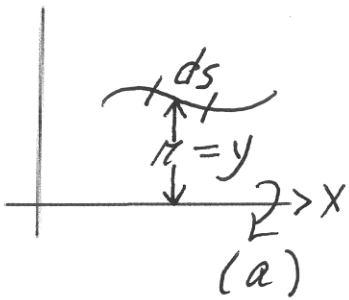
($ds$ = element of arc length)

$r$ must be expressed in terms of the variable used for $ds$.

# Example – Surface Area

Completely set up integrals for the surface area obtained by revolving $y = \sqrt{x}$ for $1 \le x \le 4$ about
(a) the x-axis, and (b) the y-axis

DIAGRAM →




(a) $dS = 2\pi r\,ds$ & $ds = \sqrt{1 + [f'(x)]^2}\,dx$

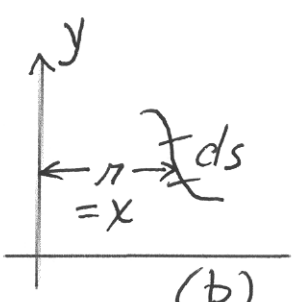
where $f(x) = \sqrt{x}$ & $f'(x) = \frac{1}{2\sqrt{x}}$

$ds = \sqrt{1 + \frac{1}{4x}}\,dx$ ← our variable is $x$

$r = y$ (from diagram) – in terms of $x$: $r = \sqrt{x}$

$dS = 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = 2\pi\sqrt{x+\frac{1}{4}}\,dx$

Answer: $S_{surf.\,area} = 2\pi\int_1^4 \sqrt{x+\frac{1}{4}}\,dx$




(b) $S = 2\pi\int_1^4 x\sqrt{1+\frac{1}{4x}}\,dx$

## Comments

- A picture of your curve is not necessary – but diagrams similar to those in the previous e.g. help with $r$.
- $r$ must be expressed in terms of the variable chosen for $ds$.
- Remember to simplify $ds$ and $dS$ before trying to integrate.