Derivatives of Polynomials and Exponential Functions

Section 3.1
Basic Differentiation Rules - 1

- If \( f(x) = c \) then \( f'(x) = 0 \).
  \[
  \frac{d}{dx} (c) = 0 \quad \text{or} \quad c' = 0
  \]
  for \( c \) constant

- If \( f(x) = x \) then \( f'(x) = 1 \).
  \[
  \frac{d}{dx} (x) = 1 \quad \text{or} \quad x' = 1
  \]

- **Power rule:** If \( f(x) = x^n \) with \( n \) constant,
  then \( f'(x) = nx^{n-1} \)
  i.e. \((x^n)' = nx^{n-1}\)
Basic Differentiation Rules - 2

- **Linearity of differentiation:**

\[ [a f(x) + b g(x)]' = a f'(x) + b g'(x) \]

when \( a \) and \( b \) are constants.

- **Consequences:**
  
  - Set \( b = 0 \) if we get \( [a f(x)]' = a f(x) \)
  
  - Set \( a = b = 1 \):

\[ [f(x) + g(x)]' = f'(x) + g'(x) \]

- Set \( a = 1, b = -1 \):

\[ [f(x) - g(x)]' = f'(x) - g'(x) \]

- Note:

\[ [a f(x) + b g(x) + c h(x)]' \]

\[ = [a f(x) + b g(x)]' + [c h(x)]' \]

\[ = a f'(x) + b g'(x) + c h'(x) \]

etc. for >3 functions.

(for \( a, b, c \ldots \) constants)
Example – Derivative of Polynomial

With linearity plus the power rule, it is easy to differentiate any polynomial.

- \[ [3x^5 - 12x^4 + 2x^2 + 5]' \]
  \[ = 3 (x^5)' - 12 (x^4)' + 2 (x^2)' + 5' \]
  \[ (\text{Linearity}) \]
  \[ = 3 (5x^4) - 12 (4x^3) + 2 (2x^1) + 0 \]
  \[ = 15x^4 - 48x^3 + 4x \]
Derivative of Exponential Function

You should know the graph of $e^x$.

**Fact:** \( \frac{d}{dx} (e^x) = e^x \)

(on \((e^x)' = e^x\))
Examples – Differentiation

- Find \( f'(x) \) when \( f(x) = 4x^4 - \frac{1}{x^2} \).

\[
f'(x) = 4 [x^4]' - 1 [x^{-2}]' = 4(4x^3) - 1(-2x^{-3}) = 16x^3 + 2x^{-3}
\]

Think of \( \frac{1}{x^2} \) as \( x^{-2} \), and use the power rule.

- Find \( g'(t) \) when \( g(t) = t\sqrt{t} + \frac{3}{t} \).

\[
g'(t) = \frac{3}{2} t^{\frac{3}{2}} + 3(-1t^{-2}) = \frac{3}{2} t^{\frac{3}{2}} - 3t^{-2}
\]

Rewrite \( g(t) = t^{3/2} + 3t^{-1} \).

- Find \( f'(x) \) when \( f(x) = 5e^x - 2 \).

\[
f'(x) = 5e^x
\]

Remember: \( (e^x)' = e^x \) and \( 2'(x) = 0 \).