Rates of Change

Section 3.3
\[ \frac{\Delta y}{\Delta x} = \text{change in } y \]
\[ \frac{\Delta x}{\Delta x} = \text{change in } x \]

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \quad (\text{or } f'(x), \text{ or slope of Tangent})
\]

So \( \frac{dy}{dx} = \text{rate of change of } y \text{ with respect to } x \)

units are \( y \)-units/\( x \)-units
Straight Line Motion

Let $s = f(t)$ be position of an object moving in a straight line.

- We know that $v(t) = \frac{ds}{dt}$ (or $f'(t)$) is its velocity – if $s$ is in meters (m) & $t$ in seconds (s), $\frac{ds}{dt}$ would be in m/s.

- When is object at rest?
  
  when velocity $= 0$
  
  we must solve $v(t) = 0$ for times at rest

- When is it moving forward (i.e. in positive direction)?
  
  when velocity $> 0$
  
  we must solve inequality $v(t) > 0$
Example of Straight Line Motion

Given $s = t^3 - 9t^2 + 24t + 2 \text{ m}$

Velocity: \( v(t) = s' = 3t^2 - 18t + 24 \text{ m/s} \)

At rest when: \( 3t^2 - 18t + 24 = 0 \)
\( 3(t^2 - 6t + 8) = 0 \)
\( 3(t - 2)(t - 4) = 0, \quad t = 2 \text{ s or } 4 \text{ s} \)

Total distance (D) traveled during first 6 seconds:

- Must count forward & backward motion

<table>
<thead>
<tr>
<th>( s )</th>
<th>( 0 )</th>
<th>( 2 )</th>
<th>( 4 )</th>
<th>( 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v &gt; 0 )</td>
<td>( v &lt; 0 )</td>
<td>( v &gt; 0 )</td>
<td></td>
<td></td>
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</tbody>
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\( D \) from \( t = 0 \) to \( t = 2 \) is \( |s(2) - s(0)| = 20 \text{ m} \)
\( D \) \( t = 2 \) to \( t = 4 \) is \( |s(4) - s(2)| = 4 \text{ m} \)
\( D \) \( t = 4 \) to \( t = 6 \) is \( |s(6) - s(4)| = 20 \text{ m} \)

Total \( D = 44 \text{ m} \)
Derivative Interpretations – 1

The problems in this section all deal with rates of change of physical quantities. Depending on your assignment, you may encounter:

• when $m = f(x)$ is mass of a thin straight rod to the left of position $x$, $\frac{dm}{dx}$ is called the linear density

• when $Q = f(t)$ represents electrical charge as a function of time, $\frac{dQ}{dt}$ is called the current

• when $N = f(t)$ is the population (# of animals or plants) at time $t$, $\frac{dN}{dt}$ is the growth rate
Derivative Interpretations – 2

• when $C = f(t)$ is the concentration of a substance in a chemical reaction, if $t$ is time, then $\frac{dC}{dt}$ is called the rate of reaction.

• when $C = f(x)$ is cost to produce $x$ units (the cost function), then $\frac{dC}{dx}$ is called the marginal cost. ("marginal" in economics refers to a derivative)