Volumes by Cross-sections

- Wanted: volume $V$ of an irregular solid

\[ V = \int_{a}^{b} A(x) \, dx \]

- Useful when you have a formula for X-sectional area $A(x)$. 
Volume of Solid of Revolution - by Disks

- Solid is generated by revolving region between curve & axis about axis
- X-sectional $A(x)$ is circular disk

Slice revolves to $dV$ of

- $dV = \pi (\text{radius})^2 \text{Thickness}$

- This setup used for variety of axes of revolution — $x$- or $y$-axis, or lines $x \ or \ y = \text{constant}$.

- $dV$ must be expressed in terms of thickness variable
Example 1 – Volume by Disks

Find the volume of the solid generated by revolving the region between \( y = x^2 \) and the \( x \)-axis, for \( 0 \leq x \leq 4 \), about the \( x \)-axis.

- draw 2-D picture

- \( \text{d} V = \pi \text{(radius)}^2 \text{d}x \)
  \[
  \text{radius} = x^2
  \]
  so \( \text{d} V = \pi x^4 \text{d}x \)

- \( V = \int_0^4 \pi x^4 \text{d}x = \pi \left[ \frac{x^5}{5} \right]_0^4 
  = \frac{1024}{5} \pi 
  \)
Volume of Solid of Revolution – by Washers

- Solid is generated by revolving region between 2 curves about non-intersecting axis

- X-sectional \( A(x) \) is washer

\[
\text{outer radius } r_{\text{out}} \quad \text{inner radius } r_{\text{in}}
\]

- \( A(x) = \pi (r_{\text{out}})^2 - \pi (r_{\text{in}})^2 \)

\[
dV = A(x) \, dx
\]

**axis of revolution might be any horiz. or vertical line.**
Example 2 – Volume by Washers

Find the volume of the solid generated by revolving the region between \( y = x^2 \) and the x-axis, for \( 0 \leq x \leq 4 \), about the y-axis.

- draw 2-D picture
- \( dV = \left[ \pi (r_{out})^2 - \pi (r_{in})^2 \right] dy \)

\( r_{out} \) is 4

\( r_{in} \) is the x-value on \( y = x^2 \) expressed in terms of thickness variable \( y \).

So \( r_{in} = \sqrt{y} \).

So \( dV = (16 \pi - \pi y) \) dy, \( y \) goes from 0 to 4

\[ V = \pi \int_{0}^{4} (16 - y) \, dy = 128 \pi \]