Area of Surface of Revolution

If we revolve a line segment about an axis (see picture), we obtain a frustrum of a cone.

Its surface area is

\[ 2\pi \text{ average length of segment} \]

We want the surface area \( S \) obtained when we revolve a smooth curve about an axis —

We must be expressed in terms of the variable used for \( ds \).
Example – Surface Area

Completely set up integrals for the surface area obtained by revolving \( y = \sqrt{x} \) for \( 1 \leq x \leq 4 \) about (a) the \( x \)-axis, and (b) the \( y \)-axis

(a) \[
\int ds = 2\pi \int ds = 2\pi \int \sqrt{1 + \left[ f'(x) \right]^2} \, dx
\]

where \( f(x) = \sqrt{x} \) \( \Rightarrow \) \( f'(x) = \frac{1}{2\sqrt{x}} \)

\[
ds = \sqrt{1 + \frac{1}{4x}} \, dx
\]

\( \text{Our variable is } x \)

\( \pi = y \) \text{ (from diagram)} – in terms of \( dx \) : \( \pi = \sqrt{x} \)

\[
\int ds = 2\pi \sqrt{x} \sqrt{1 + \frac{1}{4x}} \, dx = 2\pi \sqrt{x + \frac{1}{4}} \, dx
\]

Answer: \( S_{\text{surf. area}} = 2\pi \int_{1}^{4} \sqrt{x + \frac{1}{4}} \, dx \)

(b) \[
\int ds = 2\pi \int_{1}^{4} x \sqrt{1 + \frac{1}{4x}} \, dx
\]
Comments

• A picture of your curve is not necessary – but diagrams similar to those in the previous e.g. help with $r$.

• $r$ must be expressed in terms of the variable chosen for $ds$.

• Remember to simplify $ds$ and $dS$ before trying to integrate.